Holographic theory of topological phase transitions

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Lokman Tsui et al, Nucl. Phys. B **896**, 330 (2015).
Lokman Tsui et al, Nucl. Phys. B **919**, 470 (2017).

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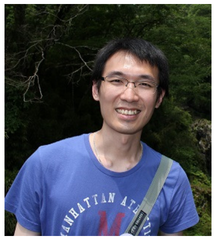
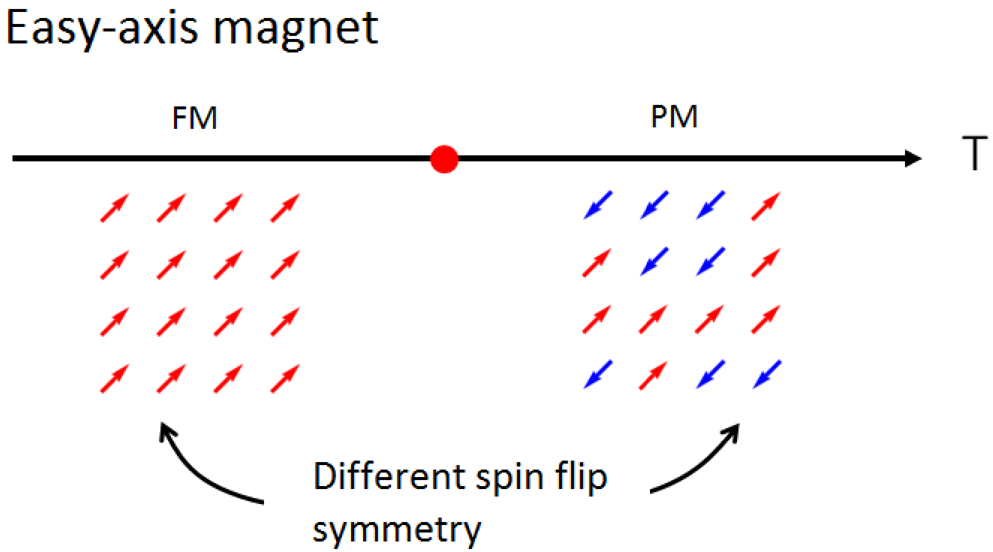


Outline

- Lecture 1 Phase transitions between bosonic SPTs and Dijkgraaf-Witten states.
- Lecture 2 Phase transitions between interacting fermionic SPTs.

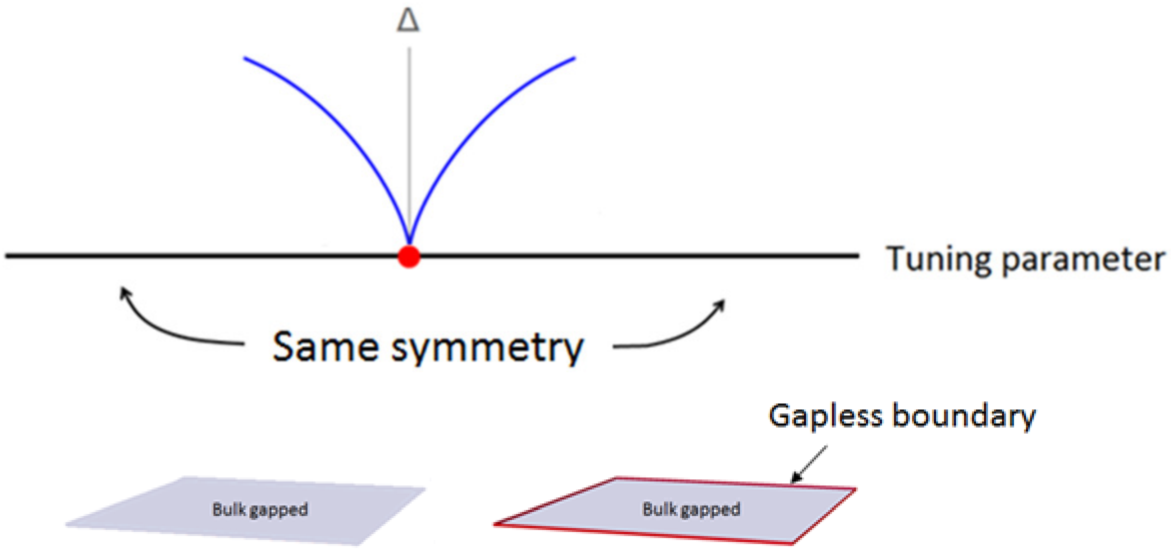
Landau transition: phase transition between phases with different symmetries



The study of Landau phase transitions has revolutionized physics.

- Renormalization group
- Conformal symmetry
- Physical understanding of continuum quantum field theories
- And many more …

Recently a new type of phase transition comes into focus. These transitions occur between gapped phases with the same symmetry.



The difference between the phases reside on their boundaries

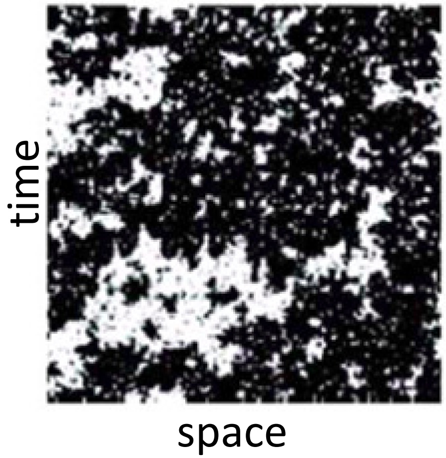
These phases are known as the symmetry protected topological states (SPTs). These are gapped phases which are not adiabatically connected so long as the symmetry of the Hamiltonian is respected.

Examples include:

- Bosonic: the Haldane phase
- Fermionic: topological insulators

Question:
What are the differences (if any) between Landau (symmetry breaking) phase transitions and the phase transitions between bosonic SPTs?

The Landau phase transitions are described by fluctuating order parameter across space-time, i.e., bosonic field theories.



To avoid obvious differences we focus on phase transition between bosonic SPTs.

The bosonic SPTs

Classification: X. Chen, Z-C Gu, Z-X Liu, X-G Wen, Science 338, 1604 (2012) . Group cohomology:

$$H^{d+1}(G, U(1))$$

An example: the Haldane phase

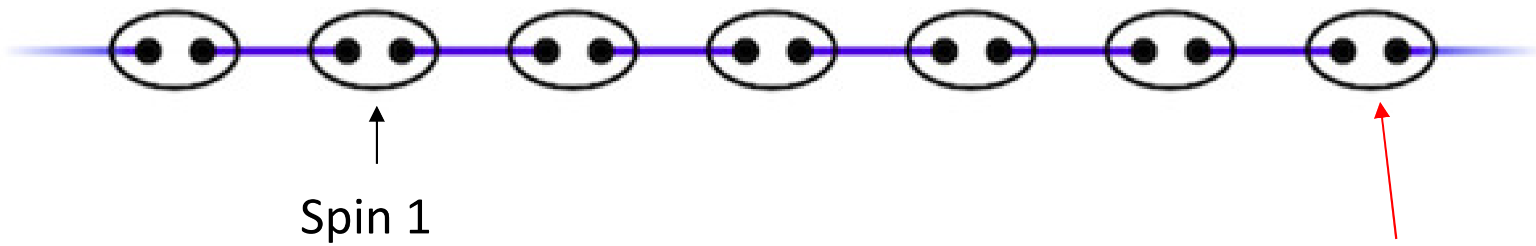
F. D. M. Haldane, Phys. Let. A 93, 464 (1983).

$$H = J \sum_{i} \vec{S}_i \cdot \vec{S}_{i+1}$$

Spin-1 AF Heisenberg model

Symmetry group SO(3)

- Bulk gapped
- Boundary gapless



Spin 1

Affleck, Kennedy, Lieb, Tasaki PRL **59**,799 (1987)

Spin 1/2

Fractionalized spin !

General classification of bosonic SPT

Given a symmetric group there is a formula predicting how many different SPTs are protected by it

X. Chen, Z-C Gu, Z-X Liu, X-G Wen, Science 338, 1604 (2012)

space dimension

The group of $e^{i\theta}$

$H^{d+1}(G, U(1))$ = an abelian group

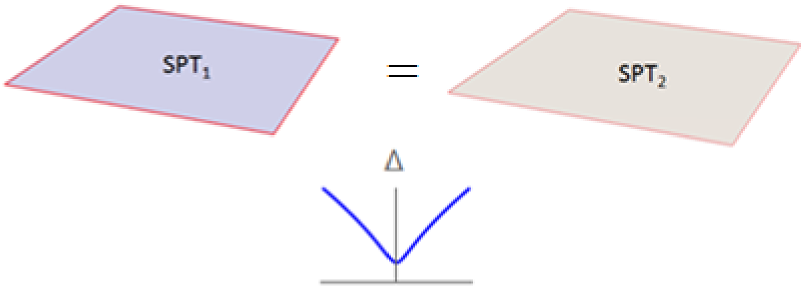
Protection symmetry group

The # of elements predicts how many different SPTs there are.

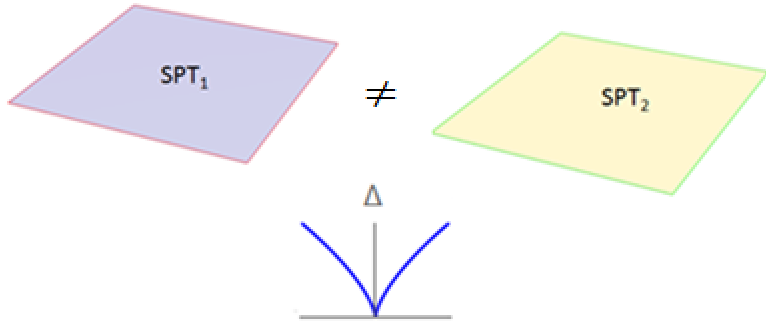
For example: $H^2(Z_n \times Z_n) = Z_n \Rightarrow$ there are n different SPT phases protected by the group $Z_n \times Z_n$ in d=1

The group structure

The equivalence relation



$$SPT_1 = SPT_2$$



$$SPT_1 \neq SPT_2$$

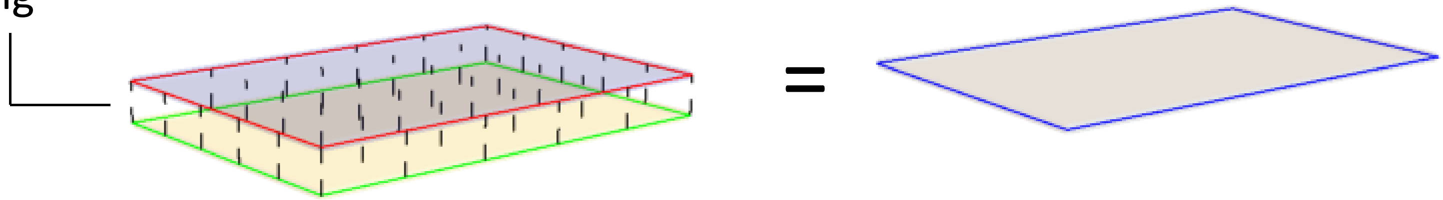
Possible to go from one phase to the other without breaking the symmetry while maintaining the energy gap.

If symmetry is preserved on a path from phase 1 to phase 2, then the energy gap must close somewhere on the way.

The "addition"

Stacking

Fully symmetric interlayer coupling



$SPT_1+SPT_2=SPT_2+SPT_1=SPT_3$ (Communitive)

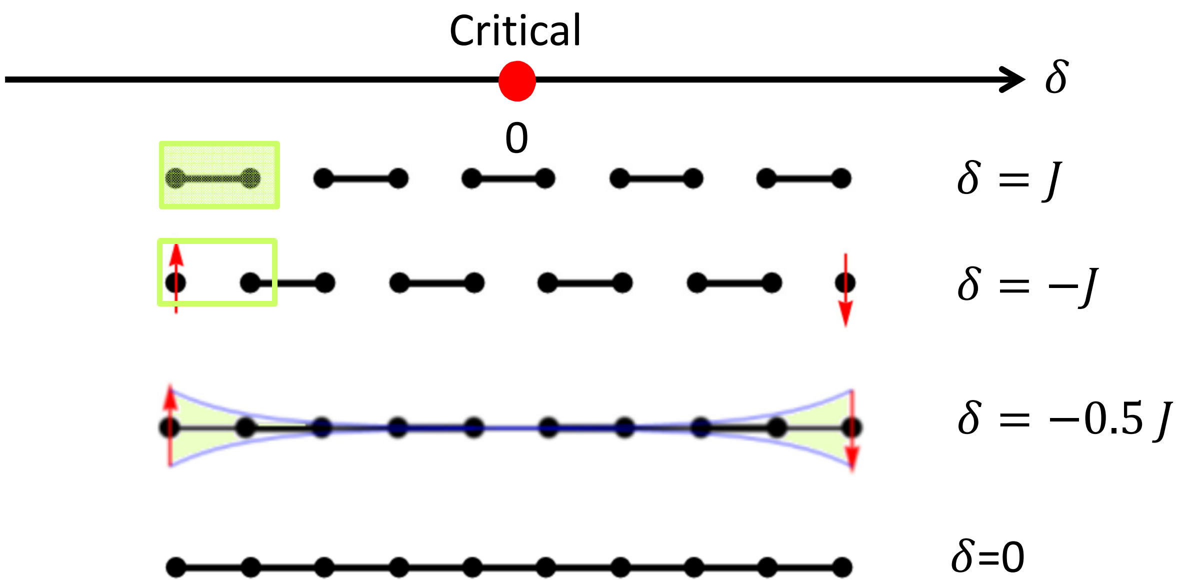
- The identity group element is the trivial SPT.
- If two SPT stacked together gives trivial SPT, these two SPT are inverse of each other.

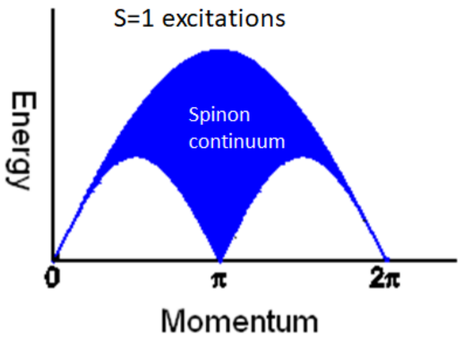
The SPT phases are elements of the group and the stacking operation is the group operation.

An example of SPT phase transition: group =SO(3)

$$H = (J-\delta)\sum_{i=even}\vec{S}_i\cdot\vec{S}_{i+1} + (J+\delta)\sum_{i=odd}\vec{S}_i\cdot\vec{S}_{i+1}$$

Spin ½ Heisenberg model with alternating bond strength

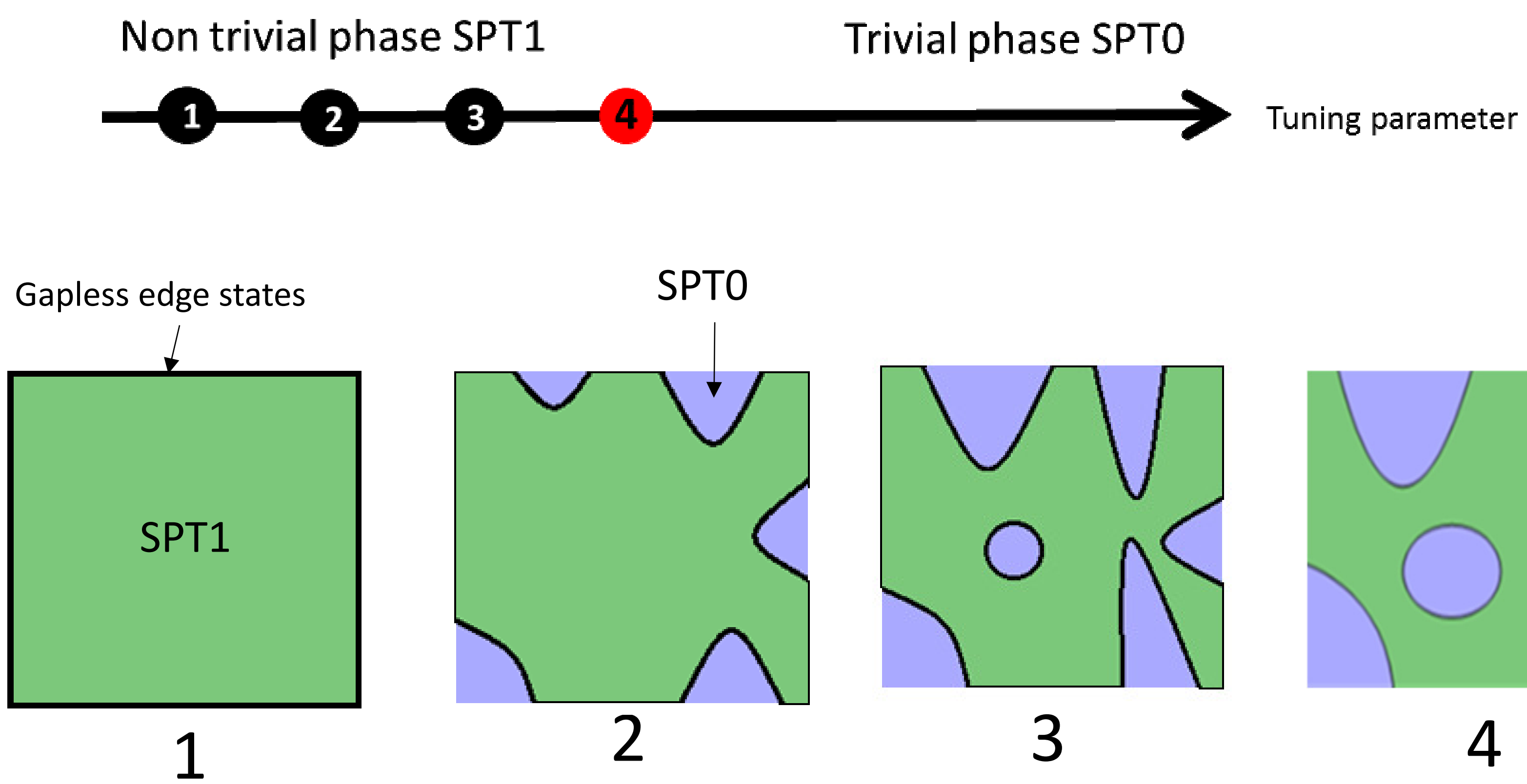




Spin ½ AF Heisenberg chain has "spinon" excitations because it is the critical theory of SPT transition. The spinons are the dynamic fluctuating boundaries of the non-trivial SPT.

The physical picture

As the system approaches criticality, the edge state become less and less localized. The probability of seeing the edge fluctuates into the bulk becomes appreciable.



⇒ Critical state consists of dynamically fluctuating boundaries of the non-trivial SPT.

A holographic theory of SPT phase transition

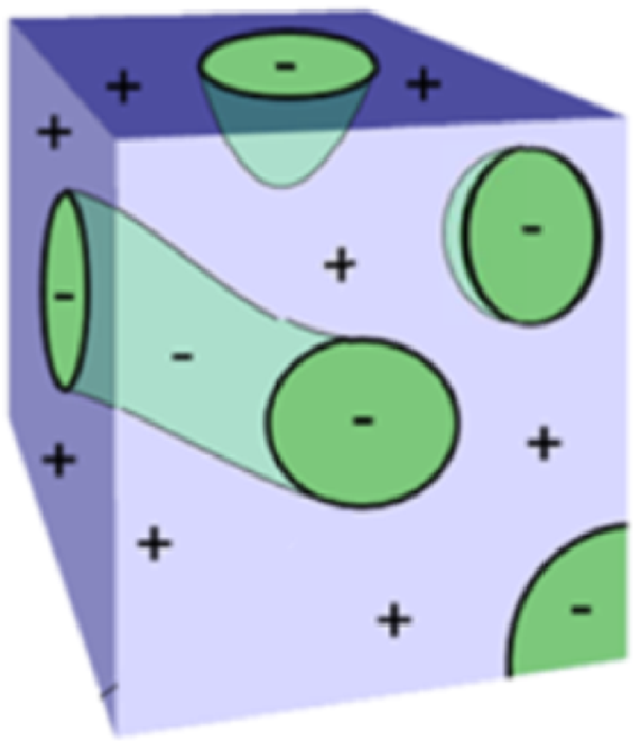
Construct a d+1 dimensional bulk SPT whose gapless boundary is the critical states of the d-dimensional SPT transition.

Lokman Tsui et al, Nucl. Phys. B **896**, 330 (2015).

How to construct the $d+1$ dimensional SPT ?

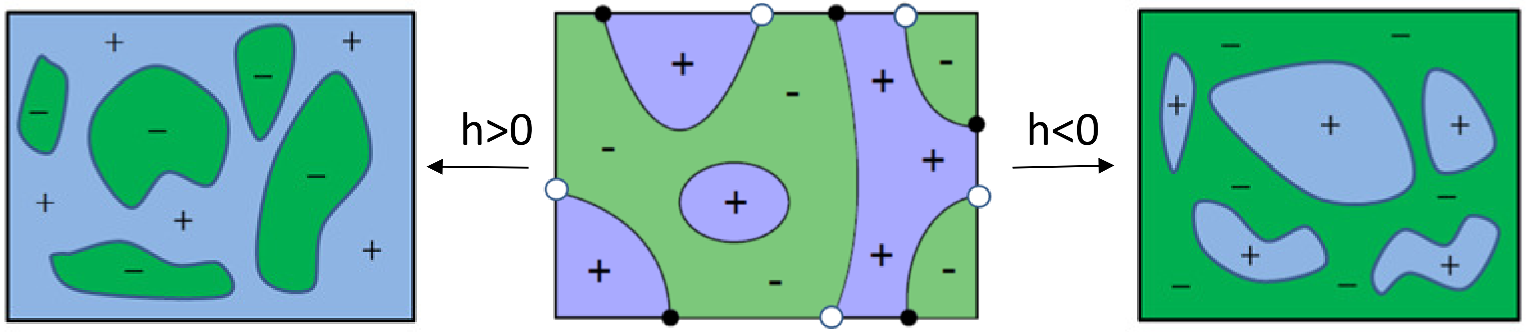
Cook up a d+1 dimensional SPT so that its boundary possesses dynamically fluctuating boundaries of the non-trivial d-dimensional SPT

$d = 2$



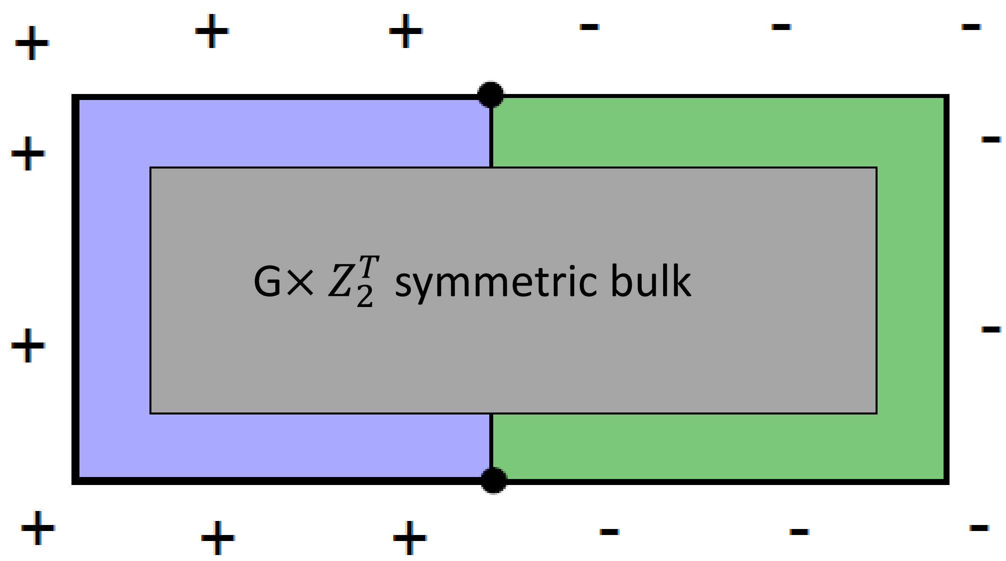
Z_2^T flips $+ \leftrightarrow -$ and changes the SPT on domain wall to the anti-SPT

Turn on explicit boundary Z_2^T symmetry breaking field



Gapless excitations disappear. But boundary is still G – symmetric.

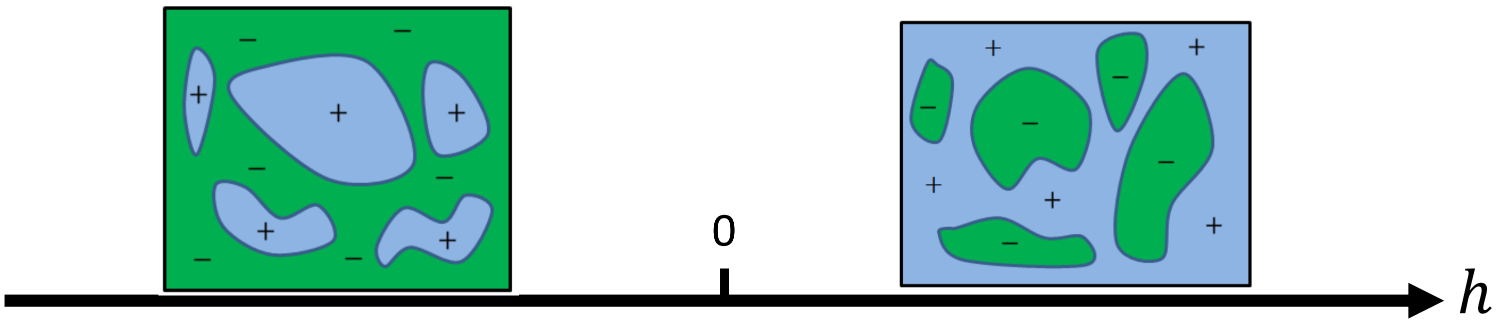
Are the two G-symmetric boundary phases induced by opposite Z_2^T symmetry breaking field inequivalent ?



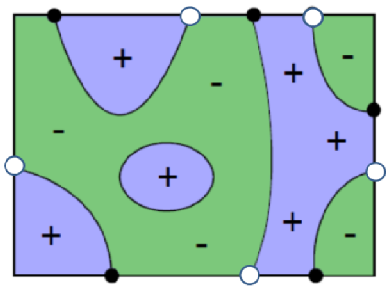
Interface must have gapless excitation because it contains the end of a domain wall.

⟹The two SPTs are inequivalent.

What are the possible states at $h = 0$?

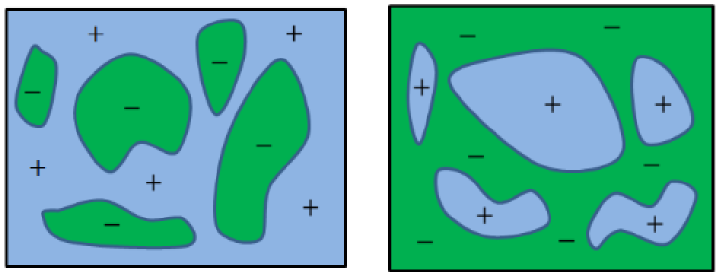


(1)



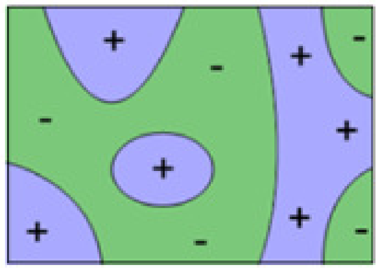
Gapless $\mathrm{G} \times Z_2^T$ symmetric

(2)



Gapped, Z_2^T symmetry is spontaneously broken

(3)



Gapped, G symmetry is spontaneously broken

(1) SPT_0 SPT_1

Continuous

$G \times Z_2^T$ symmetric

(2) SPT_0 SPT_1

First order

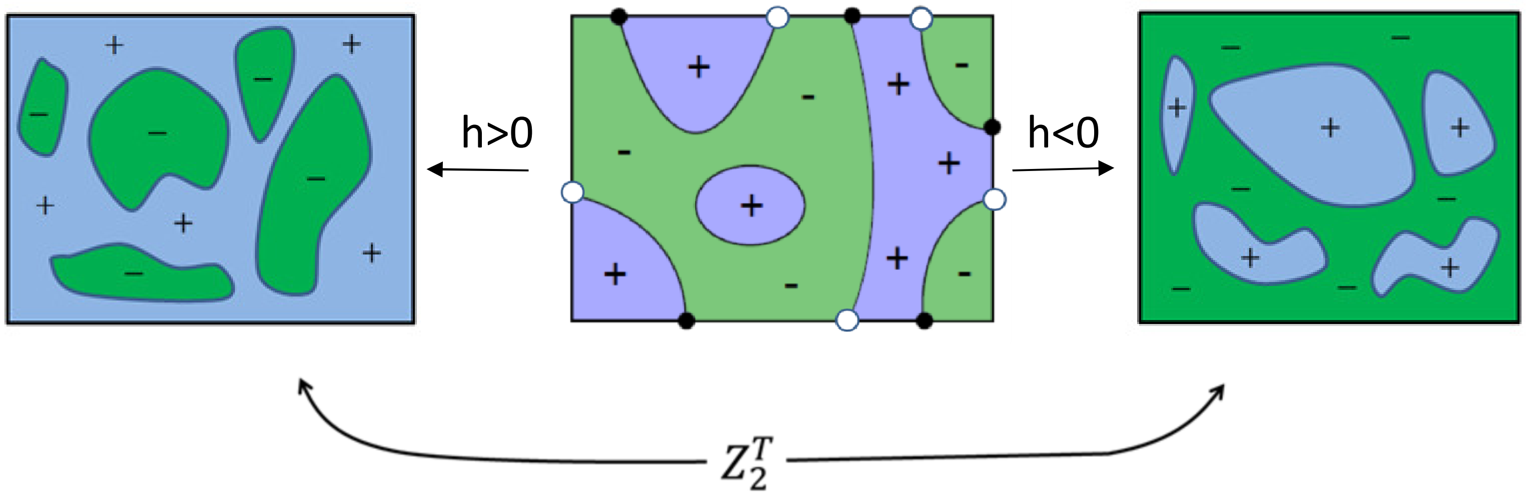
Z_2^T spontaneously broken

(3) SPT_0 SSB SPT_1

Intermediate phase

G spontaneously broken

The critical point is self-dual



Z_2^T interchanges the two SPT phases and leaves the critical point invariant. It acts as a duality transformation.

The critical state of bosonic SPT transition possesses

dynamically fluctuating boundaries of the non-trivial SPT.

Ordinary Landau transition can also have fluctuating surfaces, e.g., the domain walls of the Ising model.

However for topological phase transitions these surfaces are infested with gapless excitations.

SPT phase transitions in one dimension

Q: Which conformal field theory can describe the phase transitions between bosonic SPTs ?

Lokman Tsui et al, Nucl. Phys. B **919**, 470 (2017).

- 1D CFTs are characterized by a parameter called central charge.
- For unitary CFTs central charge is quantized when its value is less then 1.

$$c = 1 - \frac{6}{m(m+1)}; \qquad m = 3, 4, 5, \ldots$$

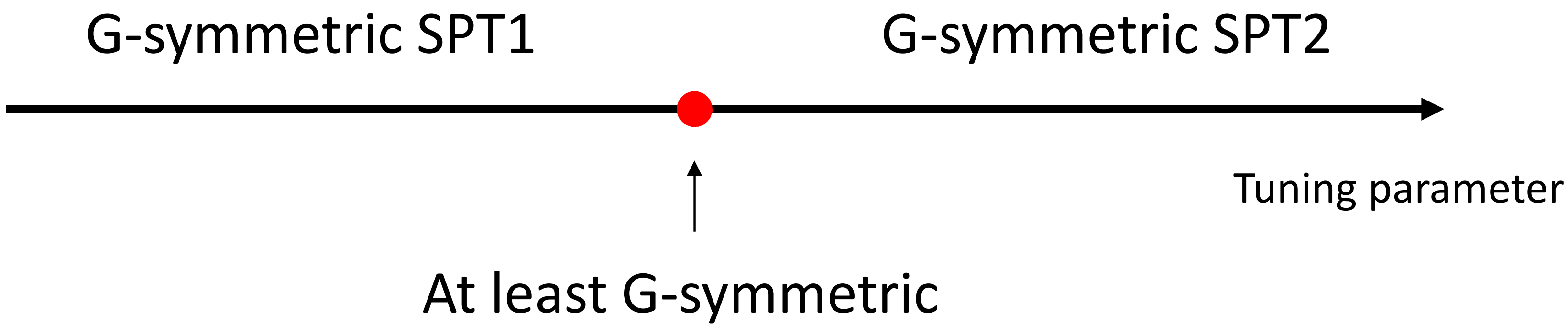
Ising   Tricritical Ising   3-state Potts

These are the "minimal models" – the best understood critical theories of Landau phase transitions.

Theorem

For minimal models the maximum internal symmetry (including the emergent symmetry) is either Z_2 or S_3.

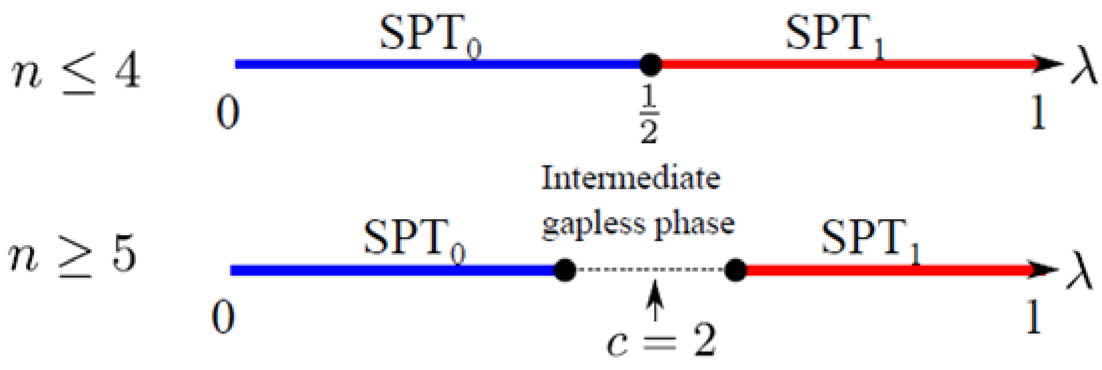
Ruelle and Verhoeven, Nucl. Phys. B 535,650 (1998)



However S_3 and Z_2 do not protect non-trivial SPT phases.

- The minimal models can not be the critical theory of the bosonic SPT phase transitions.
- The CFT of bosonic SPT transition must have $c \geq 1$

Examples: Phase transitions between $Z_n \times Z_n$ protected bosonic SPTs



Symmetry group	Central charge
$Z_2 \times Z_2$	1
$Z_3 \times Z_3$	8/5
$Z_4 \times Z_4$	2

n	$h+\bar{h}$	$h-\bar{h}$	Multiplicity
2	0	0	1
	1/4	0	2
	1	0	2+2
	1	±1	2
	5/4	±1	8

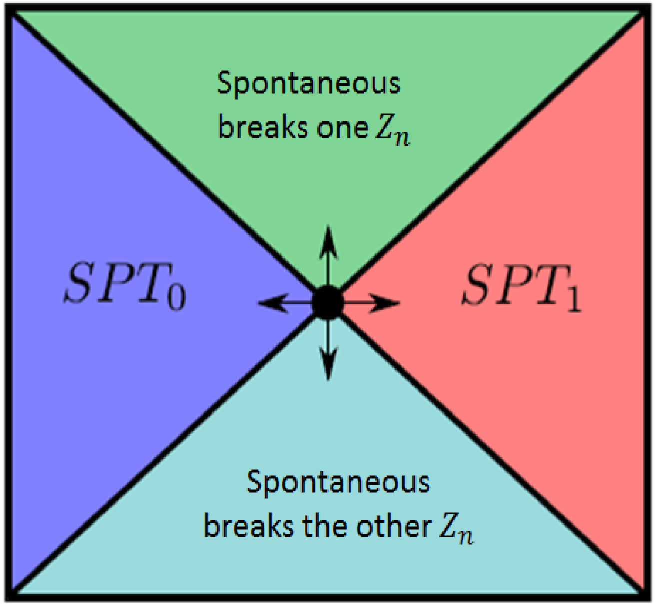
n	$h+\bar{h}$	$h-\bar{h}$	Multiplicity
3	0	0	1
	4/15	0	4
	4/5	0	2
	14/15	0	4
	17/15	±1	8
	19/15	±1	16
	4/3	0	4
	22/15	0	8
	8/5	0	1

n	$h+\bar{h}$	$h-\bar{h}$	Multiplicity
4	0	0	1
	1/4	0	4
	1/2	0	2
	5/8	0	4
	1	0	2
	9/8	±1	8
	5/4	0	20
	5/4	±1	12
	5/4	±1	16
	3/2	±1	8
	13/8	0	4
	13/8	±1	16

Entries in blue are invariant under $Z_n \times Z_n$

Blue entries in red box are gap-opening operators

But why are there two gap opening operators ?



SPT phase transitions are also Landau-forbidden transitions, i.e., they are multi-critical points.

Emergent symmetry ?

Summary

- The holographic theory $\Longrightarrow$ the critical state of bosonic SPT and Dijkgraaf Witten phases possesses dynamically fluctuating gapless boundaries of the non-trivial SPT/DW phase. The critical points are self-dual.

- In d=1 the CFT describing SPT phase transitions must have $c \geq 1$. For the models studied they are multi-critical point.

Thank you