Non-Abelian states I: Introduction

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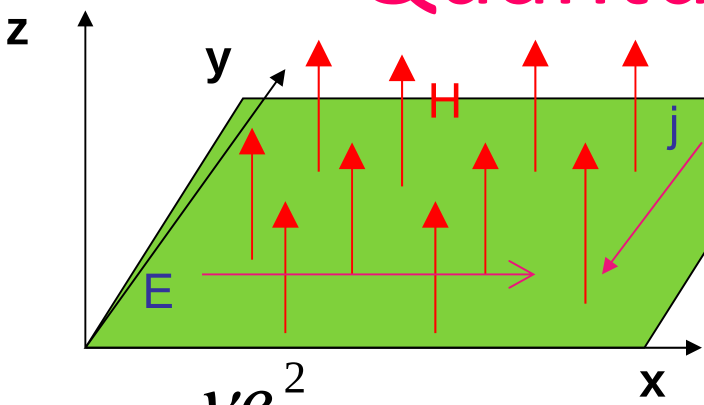
Outline

- **Quantum Hall Effect**
- **Four points of view**
  - ***- wave functions***
  - ***- topological orders***
  - ***- bulk-edge correspondence***
  - ***- composite fermions***
- **Even-denominator puzzle**
- **non-Abelian orders**
- **Topological quantum computing**
- **Numerics vs experiment**
- **What we know about $\nu = 5/2$**
- **Two possible models**
- ***- 113 order***
- ***- PH-Pfaffian order***
- **Thermal transport and non-Abelian topological order**

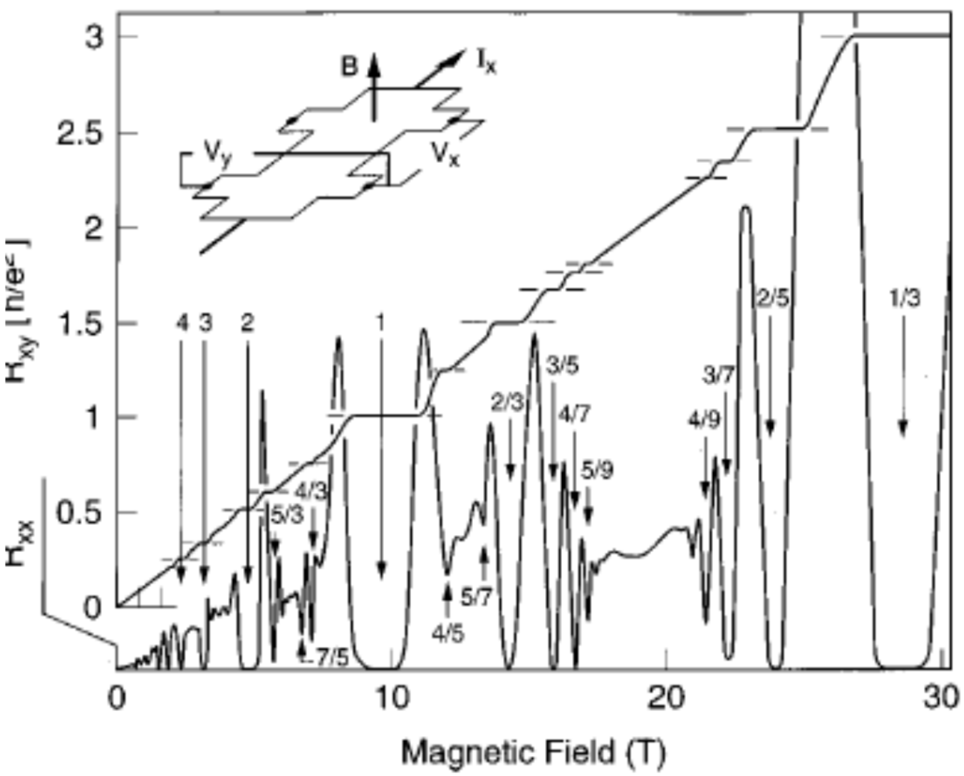
Quantum Hall Effect



$$I_x = \sigma_{xx} V = 0$$

$$I_y = \sigma_{xy} V$$

$$I = \frac{\nu e^2}{h} V; \quad \nu \text{ - rational number (filling factor)}$$



$$\frac{e^2}{h} = (25.8 k\Omega)^{-1}$$

$$= 1 \text{ Klitzing}$$

H.L. Stormer *et al*., RMP S298 (1999)

Laughlin State

$$\nu = \frac{1}{2n+1}; \quad \Psi = \prod (z_i - z_j)^{2n+1} \exp\left(-\sum |z_i|^2/4l^2\right); \; z_k = x_k + i y_k$$

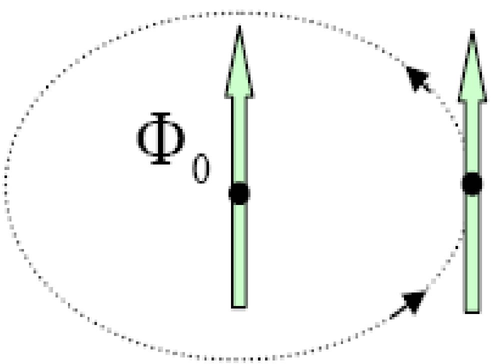
Quasiparticle charge (Laughlin)

$$q = \nu e$$

Quasiparticle statistics

(Arovas, Schrieffer, Wilczek)

$$\theta = 2\pi\nu$$



$$\theta = \frac{q}{\hbar c} \oint \vec{A} d\vec{r}$$

$(\Phi_0 = hc/e)$

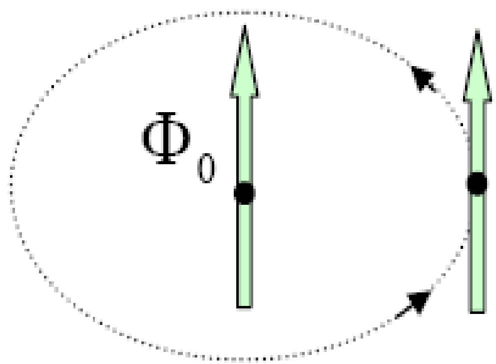
Topological order!

Wave functions

$$\nu = \frac{1}{2n+1};\ \ \Psi = \prod (z_i - z_j)^{2n+1} \exp\left(-\sum |z_i|^2/4l^2\right);\ z_k = x_k + iy_k$$

Topological order

$q = \nu e$



$\theta = 2\pi\nu$

Edge theory

$$L = -\frac{1}{4\pi\nu}\int dxdt[\partial_t\varphi\partial_x\varphi + v(\partial_x\varphi)^2]$$

Composite fermion interpretation

IQHE of (electrons+2n flux quanta)

Wave functions

Single-electron problem in a uniform magnetic field

Landau levels:

Very strong magnetic field: only the lowest Landau level matters

$$\psi = z^n \exp(-|z|^2/4l^2)$$
$$z = x + iy$$

Variational many-body wave function

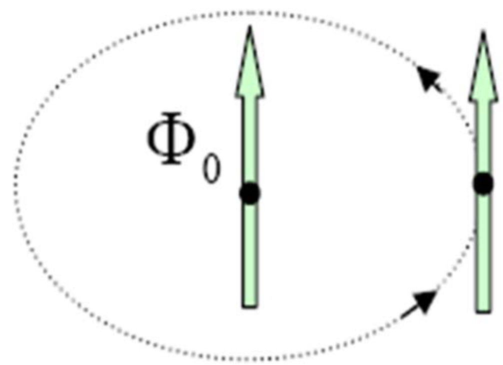
- complex analytic in all positions
- antisymmetric
- Correct charge density

Quasiparticles: flux insertion

$$\Psi \rightarrow \Psi \prod (z_i - \xi_0)$$

Statistics: Berry Phase

$$\varphi = i \int dt \left\langle \Psi[\xi(t)] \middle| \frac{d}{dt} \Psi[\xi(t)] \right\rangle$$



A puzzle of Landau-level mixing

Landau levels:

The cyclotron gap $\frac{heH}{2\pi m^* c}$ should be compared with the Coulomb energy scale $\frac{e^2}{\varepsilon l}$:

The same order of magnitude.

LLL mixing parameter ~ 1

Emergent particle-hole symmetry?

Particle-hole symmetry

Landau-level mixing breaks particle-hole symmetry.

Recent work suggests the emergent particle-hole symmetry even with Landau-level mixing:

C. Wang, N. R. Cooper, B. I. Halperin, and A. Stern, *Phys. Rev. X* **7**, 031029 (2017); G. J. Sreejith, Y. Zhang, and J. K. Jain *Phys. Rev. B* **96**, 125149 (2017).

Abelian topological order and K-matrices

Abelian state: anyon+anyon = uniquely defined anyon
statistical phases of particles add for a composite object

The K-matrix and the charge vector $\boldsymbol{t}$ contain all information about the charge and statistics of excitations, described by integer vectors $\boldsymbol{l}_k$ (X.-G. Wen, *Quantum Field Theory of Many Body Systems*):

$$\nu = \boldsymbol{t}^T K^{-1} \boldsymbol{t}$$

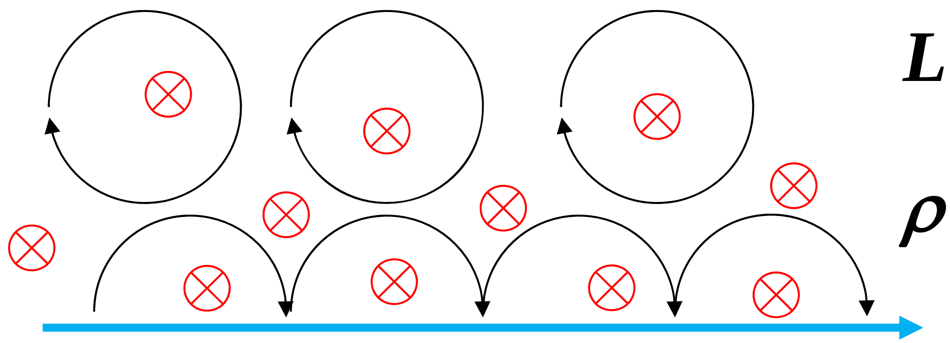
$$q_k = -e \boldsymbol{l}_k{}^T K^{-1} \boldsymbol{t}$$

$$\theta_{nk} = 2\pi \boldsymbol{l}_n{}^T K^{-1} \boldsymbol{l}_k$$

Example $\nu = 1/3$:

K =(3); $\boldsymbol{t}$=(1); $q = -\frac{e}{3}$; $\theta = \frac{2\pi}{3}$

Chiral Edge Transport



$$L = \frac{1}{4\pi}\int dtdx[\partial_t\varphi\partial_x\varphi - v(\partial_x\varphi)^2];$$

$$\rho = \partial_x\varphi/2\pi; \quad \partial_{tx}\varphi = v\partial_{xx}\varphi$$

$$\varphi = \varphi(x+vt)$$

Bulk-edge correspondence

$$L = \frac{1}{4\pi\nu}\int dydx[i\partial_y\varphi\partial_x\varphi + (\partial_x\varphi)^2]$$

$$\psi = \exp(i\varphi/\nu)$$

$$\Psi = \left\langle \hat{O}\prod \psi(z_i)\right\rangle \exp(-\sum |z_i|^2/l^2)$$

Quasiparticles?

$$\Psi = \left\langle \hat{O}\prod \psi_q\,(\xi_i)\psi(z_i)\right\rangle \exp(-\sum |z_i|^2/l^2)$$

T. H. Hansson *et al.*, *Rev. Mod. Phys.* **89**, (2017)

Composite fermions

N electrons, $\left(2p+\frac{1}{n}\right)N$ magnetic flux quanta $\frac{hc}{e}$.

Filling factor $\frac{n}{2pn+1}$: $\frac{1}{3}, \frac{2}{5}, \frac{3}{7}$ ...

Attach $2p$ flux quanta to each electron. The mutual Aharonov-Bohm phase remain trivial. Thus, *composite* particles remain *fermions*. There are only $\frac{1}{n}$ flux quanta per particle. Hence, the filling factor is the integer n for *composite fermions*.

Even-denominator filling factors

Consider the filling factor 1/2.

Attach two flux quanta to each electron. No magnetic field for composite fermions! A gapless state [Halperin-Lee-Read theory].

Alternative theory: Dirac composite fermions with particle-hole symmetry [D. T. Son, *Phys. Rev. X* **5**, 031027 (2015)]. A different physical picture. The same main prediction: no gap.

Gapped state at $\nu = 5/2$!

$\nu = 5/2$ and the Halperin 331 state

$$\Psi = \hat{A}\exp\left(-\frac{1}{4l^2}\sum[|z_{l\downarrow}|^2 + |z_{a\uparrow}|^2]\right)\prod_{k<l}(z_{k\downarrow} - z_{l\downarrow})^3 \prod_{a<b}(z_{a\uparrow} - z_{b\uparrow})^3 \prod_{k,a}(z_{k\downarrow} - z_{a\uparrow})$$

K-matrix: $K = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$ Charge vector: $\boldsymbol{t} = (1,1)$

$$\nu = \boldsymbol{t}K^{-1}\boldsymbol{t}^T = 1/2$$

2 types of anyons of charge and statistics

$$e[(K^{-1})_{11} + (K^{-1})_{12}] = \frac{e}{4}$$

$$\theta_{12} = 2\pi(K^{-1})_{12} = \frac{3\pi}{4}$$

$$\theta_{11} = 2\pi(K^{-1})_{11} = -\frac{\pi}{4}$$

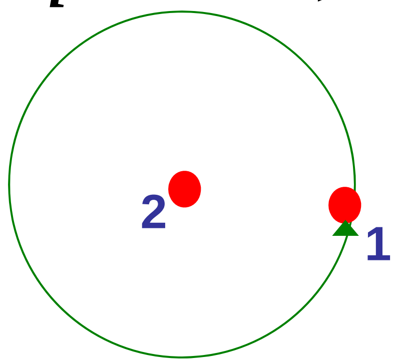
Abelian state: anyon+anyon = uniquely defined anyon

statistical phases of particles add for a composite object

Moore-Read (Pfaffian) state

$$\nu = 5/2$$

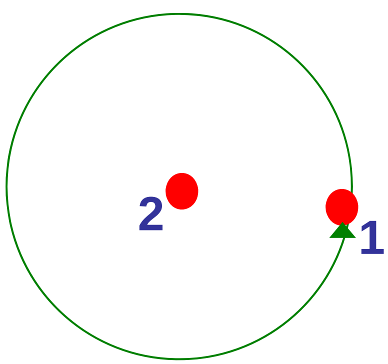
$$q = e/4;\ \text{non - Abelian statistics}$$



$$|\psi_f\rangle \neq \exp(i\theta)|\psi_i\rangle$$

Several states at given quasiparticle positions

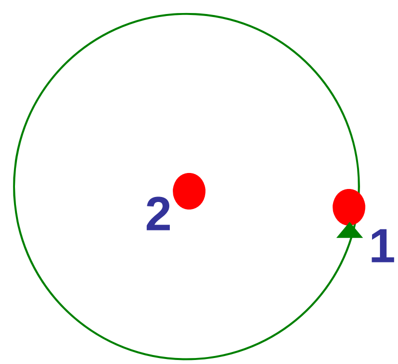
Vacuum superselection sector $|1\rangle$



$$\psi \to \psi$$

$$\theta = 0$$

Fermion sector $|\varepsilon\rangle$



$$\psi \to -\psi$$

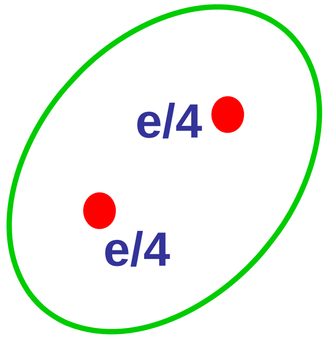
$$\theta = \pi$$

$$\alpha|1\rangle|\ \ \rangle + \beta|\varepsilon\rangle|\ \ \rangle \to \alpha|1\rangle|\ \ \rangle - \beta|\varepsilon\rangle|\ \ \rangle$$

Fusion rules

$$1\times\sigma=\sigma;\quad \varepsilon\times\sigma=\sigma;\quad \sigma\times\sigma=1+\varepsilon$$

Non-Abelian qubit and quantum computing



$$\left\{\begin{array}{l}\text{Vacuum state} = 1\\ \\ \text{Fermion state} = 0\end{array}\right.$$

Stable with respect to local perturbations!

Operations $\longrightarrow$ Quasiparticle braiding

Universal quantum computation :

12/5 (Read-Rezayi) states etc.

Bulk-edge correspondence and Pfaffian state

Edge theory: $-\frac{2}{4\pi}[\partial_t\varphi\partial_x\varphi + v_c\partial_x\varphi\partial_x\varphi] + i\psi(\partial_t + v_n\partial_x)\psi$

$\psi = \psi^+$ (Majorana fermion)

Electron operator: $\psi_e = \psi\exp(2\mathrm{i}\varphi)$

Ground state: $$\left\langle \prod \psi_e(z_i) \right\rangle \exp\left(-\frac{\sum |z_i|^2}{4l^2}\right)$$

$$= \mathrm{Pf}\left\{\frac{1}{z_i - z_j}\right\} \prod (z_i - z_j)^2 \exp\left(-\frac{\sum |z_i|^2}{4l^2}\right)$$

$$\mathrm{Pf} = \sqrt{\mathrm{Det}}$$

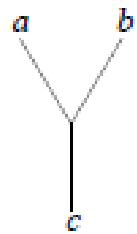
Majorana fermion emerges in 2D Ising model. What about Ising spin? Heuristically $s(z,\bar{z}) = \sigma(z)\bar{\sigma}(\bar{z})$.

$$\left\langle \prod s \right\rangle = \sum |F_k(\{z_i\})|^2$$

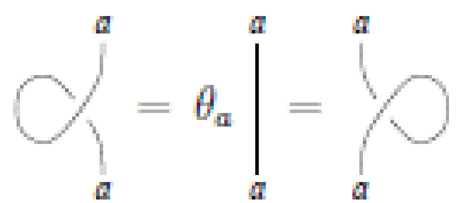
Let us view $F_k(\{z_i\})$ as correlators of $\sigma(z)$.

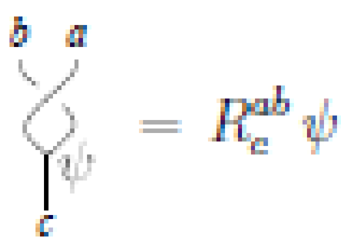
Many wave functions at fixed positions!

Key information about anyons

- Fusion rules: 

- Quantum dimension d: #states$\sim d^{\#\text{anyons}}$

- Topological spin: 

- Braiding rules: 

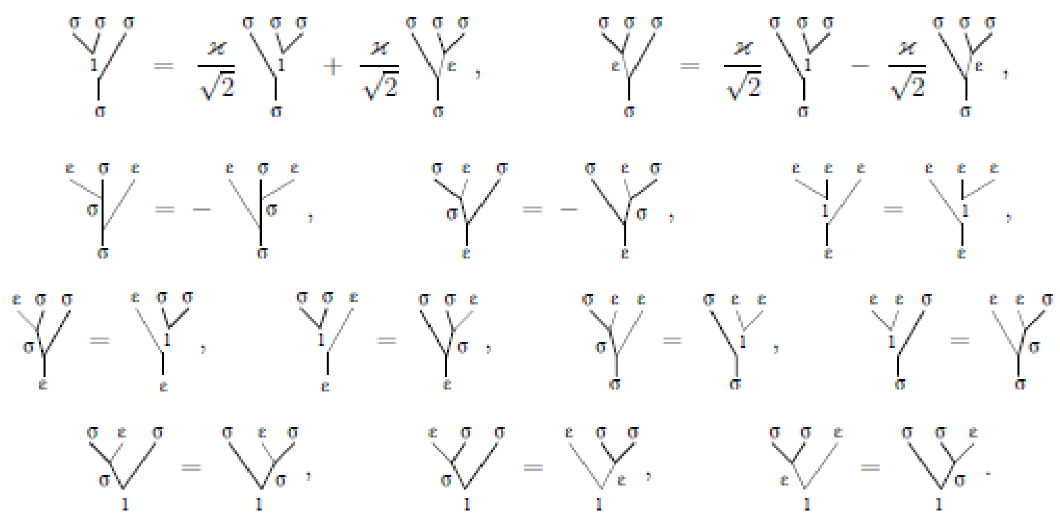
Superselection sectors: 1 (vacuum), ε (fermion), σ (vortex).

Quantum dimension:	$d_1 = 1,$	$d_\varepsilon = 1$	$d_\sigma = \sqrt{2};$
Topological spin:	$\theta_1 = 1,$	$\theta_\varepsilon = -1,$	$\theta_\sigma = \theta = \exp\left(\frac{\pi}{8} i\nu\right);$
Frobenius-Schur indicator:	$\varkappa_1 = 1,$	$\varkappa_\varepsilon = 1,$	$\varkappa_\sigma = \varkappa = (-1)^{(\nu^2-1)/8}.$

Global dimension: $\mathcal{D}^2 \stackrel{\text{def}}{=} \sum_u d_u^2 = 4.$

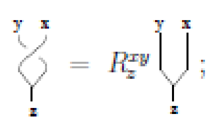
Fusion rules: $\varepsilon \times \varepsilon = 1, \qquad \varepsilon \times \sigma = \sigma, \qquad \sigma \times \sigma = 1 + \varepsilon.$

Associativity relations:



Braiding rules:

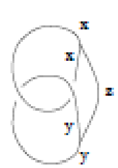
Definition of R_z^{xy}:



$$R_1^{\varepsilon\varepsilon} = -1, \qquad R_1^{\sigma\sigma} = \varkappa \exp\left(-\tfrac{\pi}{8} i\nu\right),$$

$$R_\sigma^{\varepsilon\sigma} = R_\sigma^{\sigma\varepsilon} = -i^\nu, \qquad R_\varepsilon^{\sigma\sigma} = \varkappa \exp\left(\tfrac{3\pi}{8} i\nu\right).$$

Topological S-matrix:

$(S_z)_{xy} \stackrel{\text{def}}{=} \dfrac{1}{\mathcal{D}}$  ;

$$S_1 = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \end{pmatrix}, \qquad (S_\varepsilon)_{\sigma\sigma} = \exp\left(-\tfrac{\pi}{4} i\nu\right).$$

Superselection sectors: 1 (vacuum), ε (fermion), e, m (vortices).

Quantum dimension:	$d_1 = 1,$	$d_\varepsilon = 1$	$d_e = d_m = 1;$
Topological spin:	$\theta_1 = 1,$	$\theta_\varepsilon = -1,$	$\theta_e = \theta_m = \theta = \exp(\frac{\pi}{8} i\nu);$
Frobenius-Schur indicator:	$\varkappa_1 = 1,$	$\varkappa_\varepsilon = 1,$	$\varkappa_e = \varkappa_m = \varkappa = \exp(\frac{\pi}{4} i\nu).$

Fusion rules:

$$\varepsilon \times \varepsilon = 1, \qquad \varepsilon \times e = m, \qquad \varepsilon \times m = e, \qquad e \times e = m \times m = 1, \qquad e \times m = \varepsilon.$$

<u>Case 1</u>: $\nu \equiv 0, 8 \pmod{16}$; $\qquad \theta = \pm 1,\ \varkappa = 1.$

Associativity relations: all associativity relations are trivial.

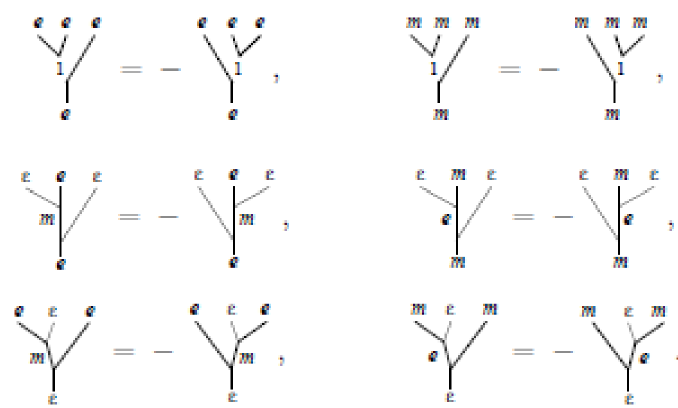
Braiding rules:

$$R_1^{\varepsilon\varepsilon} = -1, \qquad R_1^{ee} = R_1^{mm} = \theta,$$

$$R_m^{\varepsilon e} = 1, \quad R_m^{e\varepsilon} = -1, \quad R_e^{\varepsilon m} = 1, \quad R_e^{m\varepsilon} = -1, \quad R_\varepsilon^{em} = \theta, \quad R_\varepsilon^{me} = -\theta,$$

<u>Case 2</u>: $\nu \equiv \pm 4 \pmod{16}$; $\qquad \theta = \pm i,\ \varkappa = -1.$

Nontrivial associativity relations:



Braiding rules:

$$R_1^{\varepsilon\varepsilon} = -1, \qquad R_1^{ee} = R_1^{mm} = \theta,$$

$$R_m^{e\varepsilon} = R_m^{\varepsilon e} = R_e^{\varepsilon m} = R_e^{m\varepsilon} = \theta, \qquad R_\varepsilon^{em} = R_\varepsilon^{me} = 1.$$

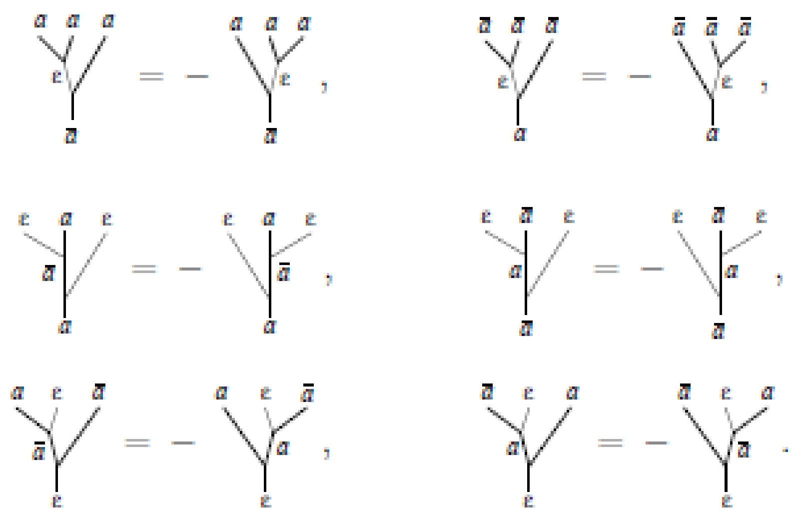
Superselection sectors: 1 (vacuum), ε (fermion), $a, \bar{a}$ (vortices).

Quantum dimension:	$d_1 = 1,$	$d_\varepsilon = 1$	$d_a = d_{\bar{a}} = 1;$
Topological spin:	$\theta_1 = 1,$	$\theta_\varepsilon = -1,$	$\theta_a = \theta_{\bar{a}} = \theta = \exp\left(\frac{\pi}{8} i\nu\right) = \frac{\pm 1 \pm i}{\sqrt{2}};$
Frobenius-Schur indicator:	$\varkappa_1 = 1,$	$\varkappa_\varepsilon = 1.$	

Fusion rules:

$$a \times \varepsilon = \bar{a}, \qquad \bar{a} \times \varepsilon = a, \qquad \varepsilon \times \varepsilon = 1, \qquad a \times a = \bar{a} \times \bar{a} = \varepsilon, \qquad a \times \bar{a} = 1.$$

Nontrivial associativity relations:



Braiding rules:

$$R_1^{\varepsilon\varepsilon} = -1, \qquad R_{\bar{a}}^{a\varepsilon} = R_{\bar{a}}^{\varepsilon a} = R_a^{\bar{a}\varepsilon} = R_a^{\varepsilon\bar{a}} = \theta^{-2},$$

$$R_\varepsilon^{aa} = R_\varepsilon^{\bar{a}\bar{a}} = \theta, \qquad R_1^{a\bar{a}} = R_1^{\bar{a}a} = \theta^{-1}.$$

"Free composite fermion" states, A. Kitaev,

Annals of Physics **321**, 2 (2006)

Summary

- Four languages for fractional quantum Hall effect
- A gapped state at an even denominator: a superconductor of composite fermions
- Non-Abelian state?
- If yes, great news for quantum computing
- Only experiment can tell

Non-Abelian states II: Particle-hole symmetry without particle-hole symmetry in the quantum Hall effect

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G. Yang and D. E. Feldman, *Phys. Rev. B* **90**, 161306(R) (2014);

P. T. Zucker and D. E. Feldman, *Phys. Rev. Lett.* **117**, 096802 (2016);

M. Banerjee, M. Heiblum, A. Rosenblatt, Y. Oreg, D. E. Feldman, A. Stern, and V. Umansky, *Nature* **545**, 75 (2017); M. Banerjee, M. Heiblum, V. Umansky, D. E. Feldman, Y. Oreg, and A. Stern, arXiv:1710.00492.

Wave functions

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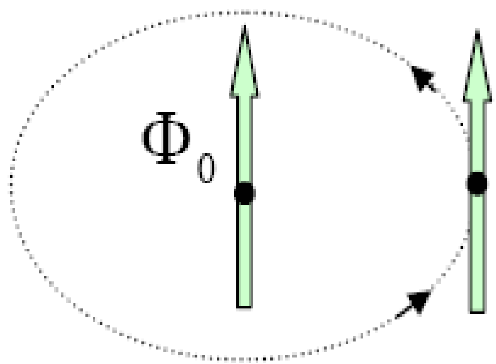
Topological order

$$q = \nu e$$



$$\theta = 2\pi\nu$$

Edge theory

$$L = -\frac{1}{4\pi\nu} \int dx dt [\partial_t \varphi \partial_x \varphi + v(\partial_x \varphi)^2]$$

Composite fermion interpretation

IQHE of (electrons+2n flux quanta)

$\nu = 5/2$ and the Halperin 331 state

Even denominator = Cooper pairs of composite fermions

$$\Psi = \hat{A}\exp(-\frac{1}{4l^2}\sum[|z_{l\downarrow}|^2 + |z_{a\uparrow}|^2])\prod_{k<l}(z_{k\downarrow} - z_{l\downarrow})^3 \prod_{a<b}(z_{a\uparrow} - z_{b\uparrow})^3 \prod_{k,a}(z_{k\downarrow} - z_{a\uparrow})$$

K-matrix: $K = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$ Charge vector: $\boldsymbol{t} = (1,1)$

$$\nu = \boldsymbol{t}K^{-1}\boldsymbol{t}^T = 1/2$$

2 types of anyons of charge $e[(K^{-1})_{11} + (K^{-1})_{12}] = \frac{e}{4}$

and statistics

$$\theta_{12} = 2\pi(K^{-1})_{12} = \frac{3\pi}{4}$$

$$\theta_{11} = 2\pi(K^{-1})_{11} = -\frac{\pi}{4}$$

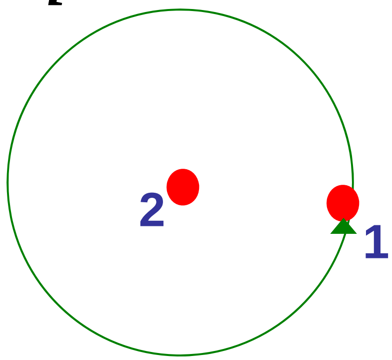
Abelian state: anyon+anyon = uniquely defined anyon

statistical phases of particles add for a composite object

Moore-Read (Pfaffian) state

$$\nu = 5/2$$

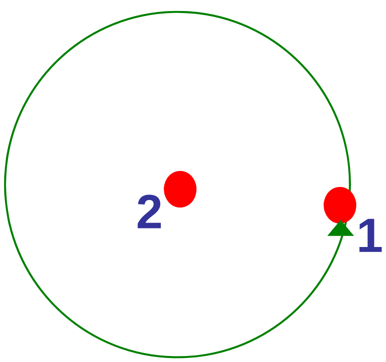
$$q = e/4; \text{non - Abelian statistics}$$



$$\left|\psi_f\right\rangle \neq \exp(i\theta)\left|\psi_i\right\rangle$$

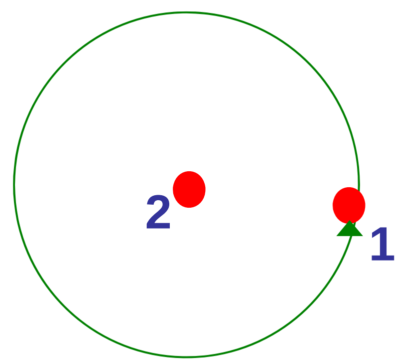
Several states at given quasiparticle positions

Vacuum superselection sector $\left|1\right\rangle$ Fermion sector $\left|\varepsilon\right\rangle$



$$\psi \rightarrow \psi$$

$$\theta = 0$$



$$\psi \rightarrow -\psi$$

$$\theta = \pi$$

$$\alpha\left|1\right\rangle\left|\quad\right\rangle + \beta\left|\varepsilon\right\rangle\left|\quad\right\rangle \rightarrow \alpha\left|1\right\rangle\left|\quad\right\rangle - \beta\left|\varepsilon\right\rangle\left|\quad\right\rangle$$

Theoretical proposals at ν=5/2

Numerous ways to build Cooper pairs!

- Pfaffian state
- 331 state
- K=8 state
- $SU(2)_2$ state
- anti-Pfaffian state
- anti-331 state
- anti- $SU(2)_2$ state
- and so on

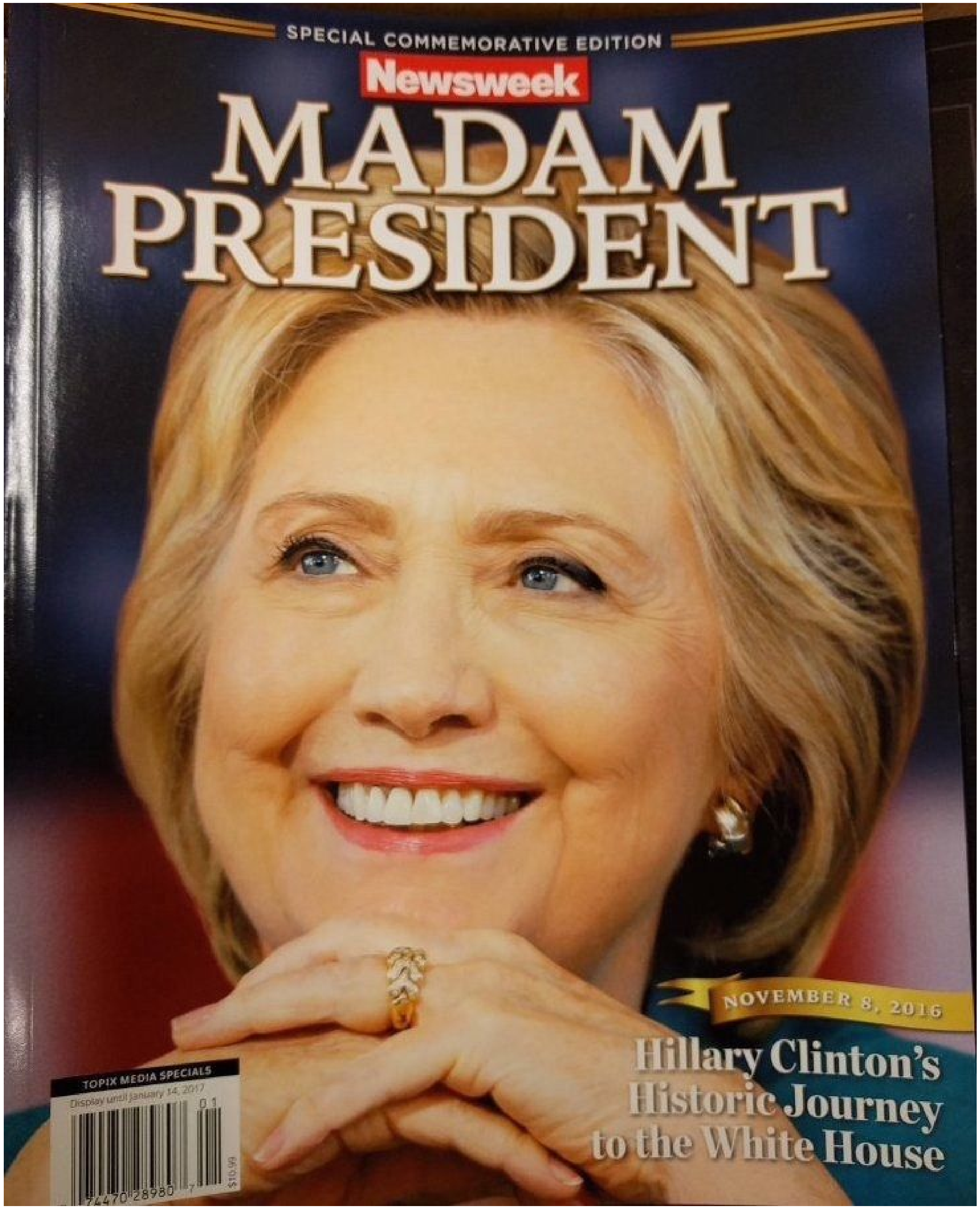
Numerics favors Pfaffian and anti-Pfaffian

Pfaffian and anti-Pfaffian states

- numerics supports Pfaffain and anti-Pfaffian states in the absence of disorder and Landau level mixing
- poor results for the energy gap
    - strong disorder
    - LLM parameter ~ 1.3
- Small energy differences for proposed states

  [J. Biddle *et al.*, *Phys. Rev. B* **87**, 235134 (2013)]
- no QHE at realistic LLM in numerics

[K. Pakrouski *et al.*, *Phys. Rev. X* **5**, 021004 (2015);
E. H. Rezayi, *Phys. Rev. Lett.* **119**, 026801 (2017)]


SPECIAL COMMEMORATIVE EDITION
Newsweek
MADAM PRESIDENT
NOVEMBER 8, 2016
Hillary Clinton's Historic Journey to the White House
TOPIX MEDIA SPECIALS
Display until January 14, 2017

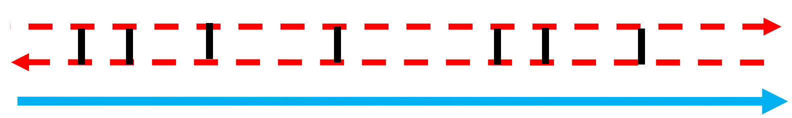
Experimental results

- Charge **e/4**
- Controversial results for spin polarization
- Fabry-Perot interferometry identical for all non-Abelian states and can be identical for Abelian states [**A. Stern, B. Rosenov, R. Ilan, and B.I. Halperin, PRB 82, 085321 (2010)]**

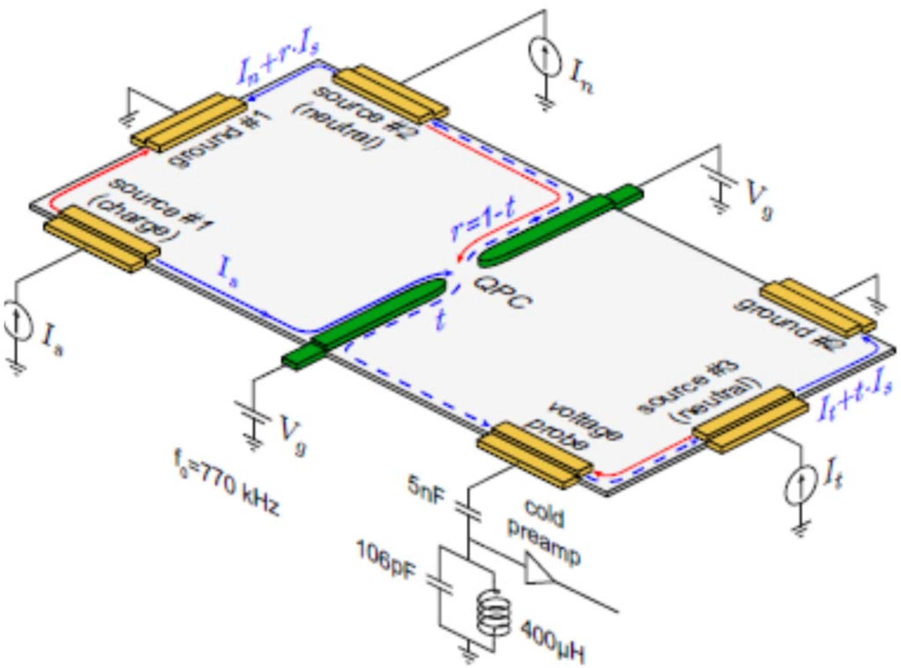
Upstream modes

Topologically protected

Edge reconstruction



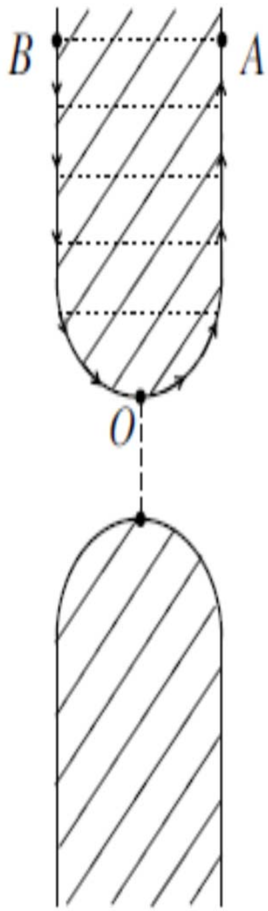
Upstream neutral modes



Observation of a topologically protected upstream neutral mode [A. Bid *et al.,* Nature **466**, 585 (2010); M. Dolev *et al*., PRL **107**, 036805 (2011)]. Compatible with anti-Pfaffian. Incompatible with Pfaffian.

The observed physics at ν=5/2 is similar to ν=8/3. It differs from the filling factors such as 7/3 where no topologically protected upstream modes are present but edge reconstruction is possible [H. Inoue *et al*., Nature Comm. **5**, 4067 (2014).]

Tunneling



Theory: $G \sim T^{2g-2}$

- Pfaffian: $g = \frac{1}{4}$
- Anti-Pfaffian: $g = \frac{1}{2}$

Experiment: $g_{\mathrm{exp}} > g_{\mathrm{theor}}$

Experiment gives an upper bound on g
The upper bound of 0.4 is consistent with Pfaffian and excludes anti-Pfaffian

I. P. Radu *et al.*, *Science* **320**, 899 (2008); X. Lin *et al.*, *Phys. Rev. B* **85**, 165321 (2012); S. Baer *et al.*, *Phys. Rev. B* **90**, 075403 (2014); H. Fu *et al.*, *PNAS* **113**, 12386 (2016).

Filling factor 1/2

[D. T. Son, *Phys. Rev. X* **5**, 031027 (2015)]

Imagine exact particle-hole symmetry between filling factors f and 1-f.

Geometric resonance experiments hint at such symmetry.

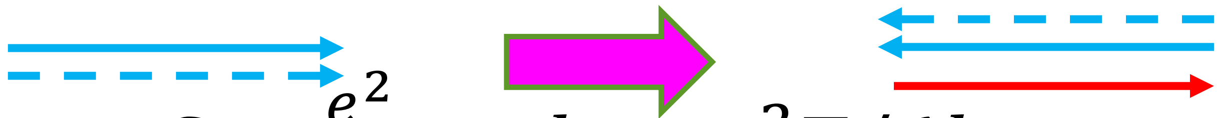
The theory can be made explicitly symmetric by assuming that composite fermions are Dirac particles.

What about Cooper pairing of Dirac particles in the *s*-channel?

PH-Pfaffian state

s-pairing of Dirac fermions

Particle-hole symmetry:



$$G = \frac{e^2}{2h}; \qquad k = \pi^2 T/6h$$

Edge theory:

[see also S.-S. Lee *et al.*, *PRL* **99**, 236807 (2007)]

$$-\frac{2}{4\pi}[\partial_t\varphi\partial_x\varphi + v_c\partial_x\varphi\partial_x\varphi] + i\psi(\partial_t - v_n\partial_x)\psi$$

$$\psi = \psi^+$$

Wave function:

$$\int\{d^2 s_i\}\mathrm{Pf}\left\{\frac{1}{\bar{s}_i - \bar{s}_j}\right\}\prod(s_i - s_j)^2 \exp[-2|s_i|^2 + 2\bar{s}_i z_i - |z_i|^2]$$

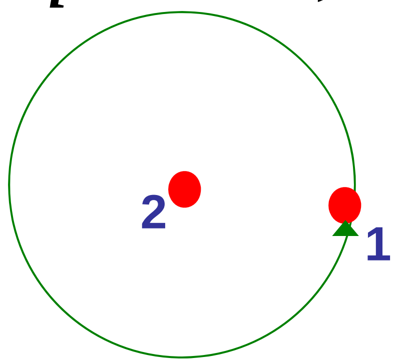
The same topological order in a different context:

L. Fidkowski *et al.*, *PRX* **3**, 041016 (2013); P. Bonderson *et al.*, *J. Stat. Mech.* P09016 (2013)

PH-Pfaffian state

$$\nu = 5/2$$

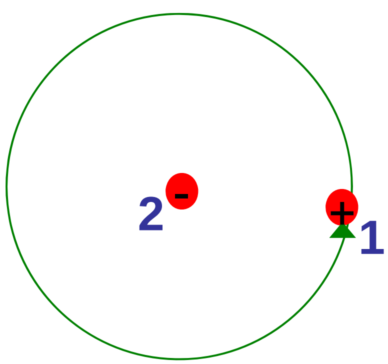
$$q = e/4;\ \text{non - Abelian statistics}$$



$$\left|\psi_f\right\rangle \neq \exp(i\theta)\left|\psi_i\right\rangle$$

Several states at given quasiparticle positions

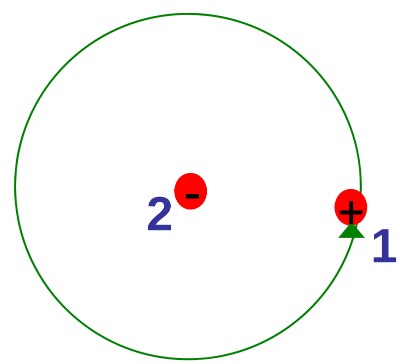
Vacuum superselection sector $\left|1\right\rangle$



$$\psi \to \psi$$

$$\theta = \pi/2$$

Fermion sector $\left|\varepsilon\right\rangle$



$$\psi \to -\psi$$

$$\theta = -\pi/2$$

$$\alpha\left|1\right\rangle\left|\ \right\rangle + \beta\left|\varepsilon\right\rangle\left|\ \right\rangle \to i\alpha\left|1\right\rangle\left|\ \right\rangle - i\beta\left|\varepsilon\right\rangle\left|\ \right\rangle$$

Fusion and braiding rules

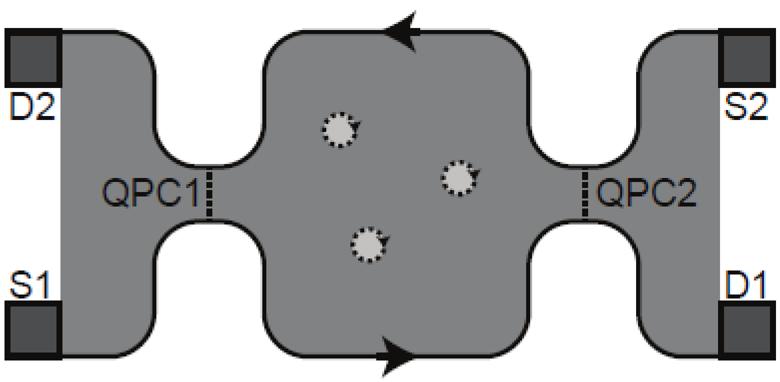
- 6 quasiparticle types:

  topological charge σ and electric charges $\pm e/4$;

  topological charges 1 and ψ and electric charges 0 and $e/2$
- Fusion rules $\psi \times \psi = 1; \psi \times \sigma = \sigma; \sigma \times \sigma = 1 + \psi$
- Braiding rules determine the phase accumulated by a quasiparticle moving around another quasiparticle. The phase depends on the fusion channel.

Comparison with the experiment

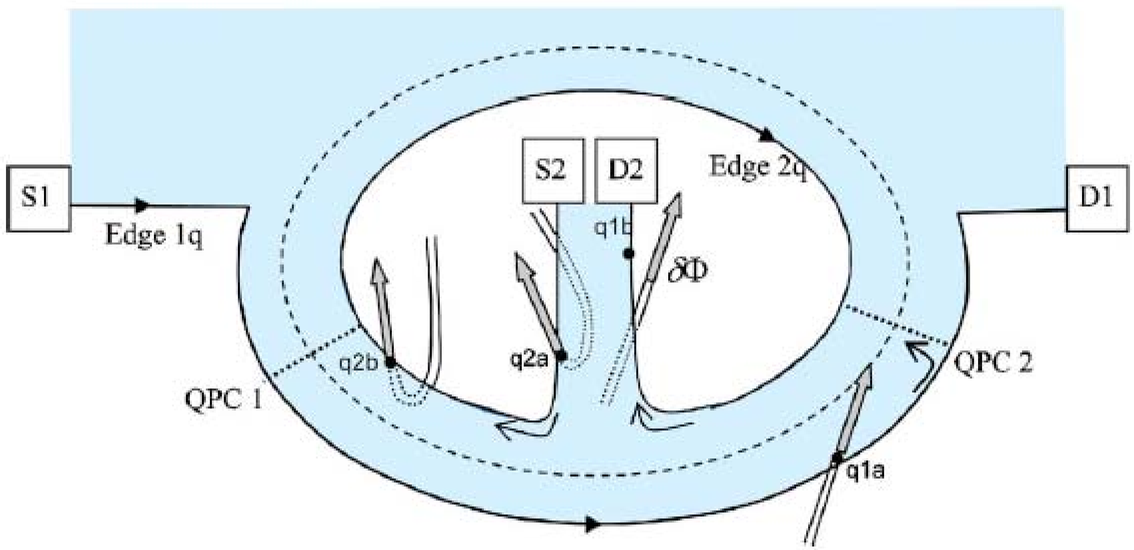
- An upstream neutral mode
- Tunneling exponent $g = \frac{1}{4}$
- Topological even-odd effect



New experimental signatures

Thermal Hall conductance $\frac{\pi^2 k^2 T^2}{6h}$

Mach-Zehnder interferometry

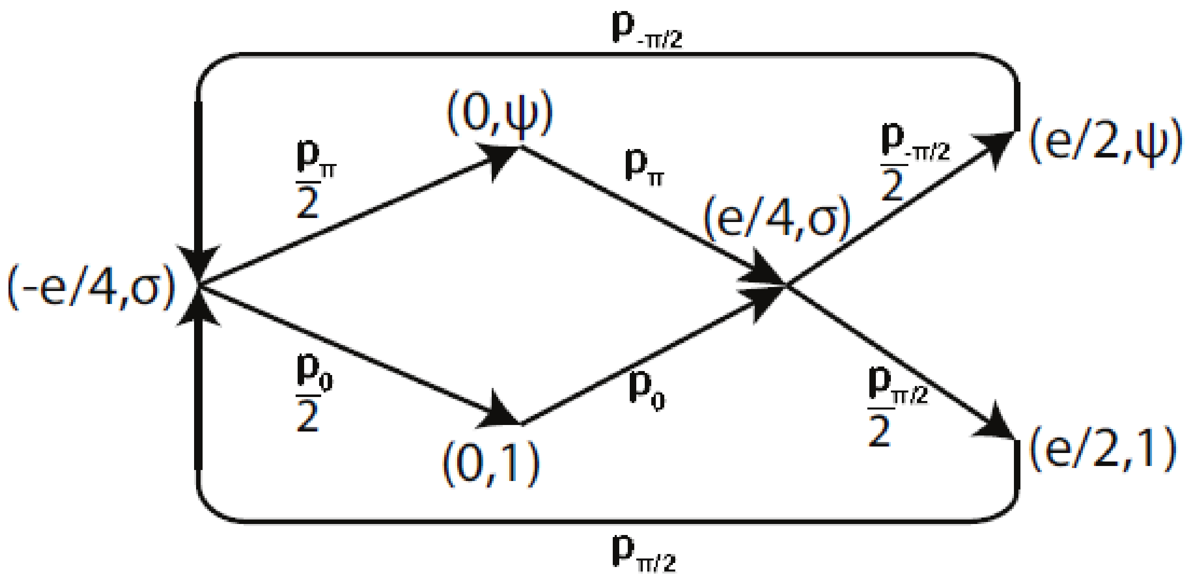


No magnetic field dependence of the current.
Shot noise diverges at some magnetic fields.

Mach-Zehnder interferometry

Tunneling probability depends on the accumulated topological charge

$$P \sim |\Gamma_1|^2 + |\Gamma_2|^2 + 2u|\Gamma_1\Gamma_2|\cos(\varphi_{AB} + \varphi_{stat} + \alpha)$$

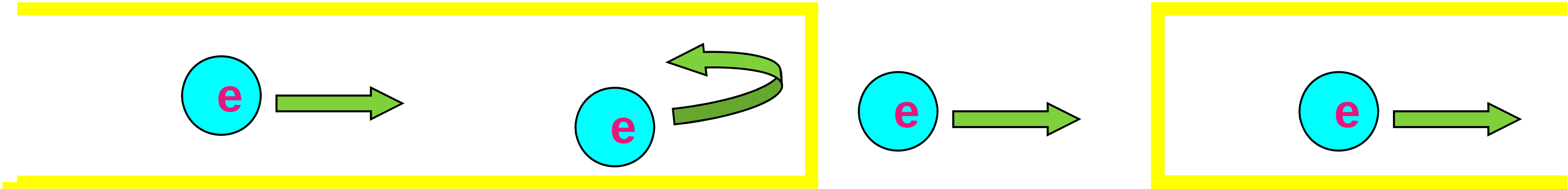


$$I = \text{const}[|\Gamma_1|^2 + |\Gamma_2|^2]$$

Shot Noise

$$S = \int [\langle I(0)I(t)\rangle - \langle I\rangle^2]dt = q\langle I\rangle$$

Walter Shottky



L. Saminadayar et al., R. de Picciotto et al. (1997):q=e/3

Shot noise in MZ interferometer

$$S = q^*\langle I\rangle$$

$$q^* = \frac{e}{64}\sum p_i \sum \frac{1}{p_i} \text{ diverges at some fields}$$

P. T. Zucker and D. E. Feldman, PRL **117**, 096802 (2016)

113 state

$$\Psi = \hat{A}\exp\left(-\frac{1}{4l^2}\sum\left[|z_{l\downarrow}|^2 + |z_{a\uparrow}|^2\right]\right)\prod_{k<l}\left(\partial_{z_{k\downarrow}} - \partial_{z_{l\downarrow}}\right)^2 \prod_{a<b}\left(\partial_{z_{a\uparrow}} - \partial_{z_{b\uparrow}}\right)^2 \times$$

$$\prod\nolimits_{k<l}(z_{k\downarrow} - z_{l\downarrow})^3 \prod\nolimits_{a<b}(z_{a\uparrow} - z_{b\uparrow})^3 \prod\nolimits_{k,a}(z_{k\downarrow} - z_{a\uparrow})^3$$

Similar to the 112 state at $\nu = 2/3$: X. G. Wu *et al.*, PRL **71**, 153 (1993); I. A. McDonald and F. D. M. Haldane, PRB **53**, 15845 (1996).

K-matrix: $K = \begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix}$

2 types of anyons

$$\theta_{12} = 2\pi (K^{-1})_{12} = \frac{3\pi}{4}$$

$$\theta_{11} = 2\pi (K^{-1})_{11} = -\frac{\pi}{4}$$

G. Yang and D. E. Feldman, PRB **90**, 161306 (2014)

Transport

$$L = -\frac{1}{4\pi}\int dtdx\,[2\partial_t\varphi_c\partial_x\varphi_c - \partial_t\varphi_n\partial_x\varphi_n + 2v_c(\partial_x\varphi_c)^2 + v_n(\partial_x\varphi_n)^2 + 2v_{cn}\partial_x\varphi_c\partial_x\varphi_n]$$

φ_c and φ_n are charged and neutral modes;
v_c and v_n are their speeds, and v_{cn} is their interaction strength;
charge density $\rho = e\,\partial_x\varphi_c/2\pi$

√ The minus sign in the action reflects an upstream mode

?

$$g_\pm = \frac{1}{\sqrt{1-c^2}}\left(\frac{3}{8} \pm \frac{c}{2\sqrt{2}}\right);\ c = \frac{\sqrt{2}v_{cn}}{v_c+v_n}$$

$c = 0$ would agree well with the experimental $g = 0.4$. But $c \neq 0$!

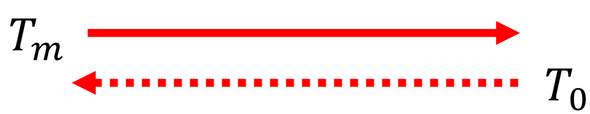
√ Explanation: $v_c \gg v_n, v_{cn}$ because the charged mode participates in long-range Coulomb interaction and the neutral mode does not. Thus $c \ll 1$.

Thermal transport

Thermal conductance of a bosonic chiral channel
$\kappa = 1$ in units of $\pi^2 k^2 T/3h$

Thermal conductance of a Majorana channel
$\kappa = 1/2$ in units of $\pi^2 k^2 T/3h$

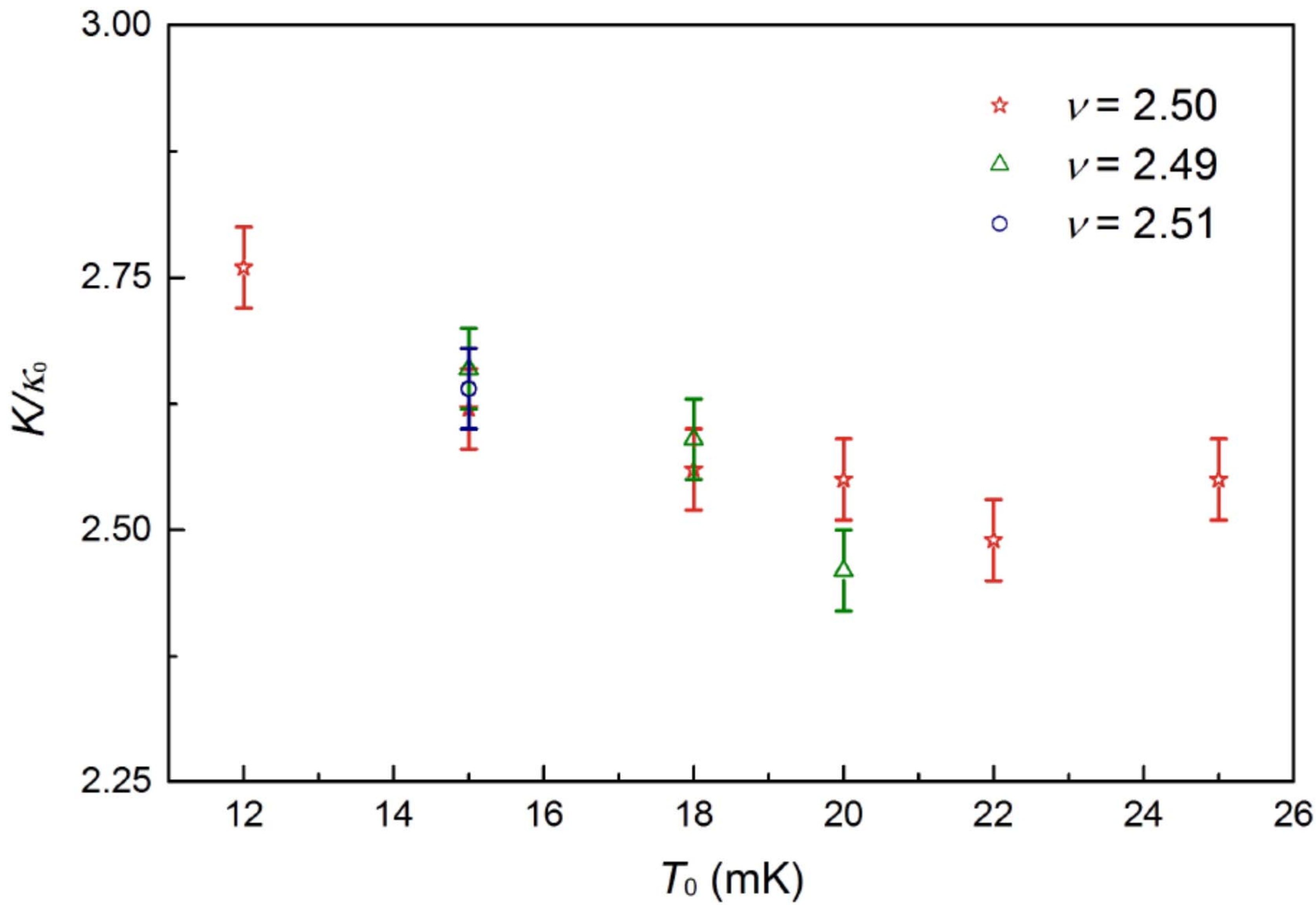
Excellent agreement of experiment and theory at f=1/3, 3/5, 4/7
[M. Banerjee, M. Heiblum, A. Rosenblatt, Y. Oreg, D. E. Feldman, A. Stern, and V. Umansky, *Nature* **545**, 75 (2017)]



At $f = 2/3$, $\kappa = 0.25 - 0.33$.
Equilibration of two channels at different temperatures, $\kappa \sim \xi/L$.
Equilibration length ξ is longer at lower temperatures.
At $f = 5/2$ one expects a faster grows of ξ at low T in the PH-Pfaffian state but κ depends on $\exp(-\mathrm{const}\, L/\xi)$.

Thermal conductance at f=5/2

M. Banerjee, M. Heiblum, V. Umansky, D. E. Feldman, Y. Oreg, and A. Stern, arXiv:1710.00492



Closing Argument

- PH-Pfaffian topological order is consistent with all experiments
- Numerics with the particle-hole symmetric Hamiltonians supports states that break the particle-hole symmetry
- Realistic Hamiltonians have no symmetry
- The ground state is not symmetric, yet the topological order is compatible with the particle-hole symmetry