Energy dependence of impact fragmentation of long glass rods

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Abstract

We have performed an experimental study of impact fragmentation with a focus on the dependence on energy input. Long glass rods were dropped horizontally onto the ground from seven different heights. We find that the energy dependence is better characterized by studying the differential mass distribution rather than the cumulative mass distribution. For lower dropping heights, the differential mass distribution is well approximated by one power law while for higher heights, it has to be represented by two power laws. Moreover, the power-law exponent for small mass fragments increases and approaches an asymptotic value as the dropping height is increased. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

Fragmentation is a physical process that occurs commonly in everyday life and in many areas of science and technology. Examples include breaking of glasses at home and breaking of raw ore in mineral processing. In all situations, an object breaks up into many smaller pieces or fragments as a result of some external impact forces that impart a sufficient amount of energy. In some cases we wish to prevent fragmentation from happening at all while in other cases we would like the object to be fragmented as thoroughly as possible. Fragmentation processes are complex and involve propagation of many cracks and their interaction. As a result, it is of interest to study the overall statistical aspects.

One of the remarkable statistical features observed is the power-law distribution of the fragment mass. For example, in an experimental study of brittle fracture of glass...
with the power-law exponent $\alpha \approx \frac{2}{3}$. Oddershede et al. [2] carried out an experimental study of fragmentation of objects made of different materials which include gypsum, soap, and potato. They found that $F(m)$ is insensitive to the types of the material studied and can be fitted by a power law with an exponential cutoff. The power-law exponent $\alpha$ was found to depend on the shape or the effective dimension of the object. In a later study by Meibom and Balslev [3], who investigated fragmentation of plates of dry clay, $F(m)$ was instead found to have two power-law regions with the exponents for large and small fragments corresponding, respectively to those of two and three dimensions. Thus, the power-law exponent depends on the dimensionality of the original object on the scale of the fragment considered and not simply on the global dimensionality of the object. These works lead to the natural query in the universality of the fragment mass distributions in impact fragmentation.

Aspect ratio dependence of the fragment mass distribution was studied in a numerical model of three-dimensional impact fragmentation [4]. The model gives $F(m)$ which shows a power law with a flat tail when a rectangular parallelepiped object is hit on its thin side but a power law with a cutoff when it is hit by its broad side. Energy dependence of the mass distribution has been reported in a recent numerical study of fragmentation of two-dimensional solids using molecular dynamics calculations by Ching et al. [5]. In that work, an object was modeled by a set of particles which interact pairwisely via a truncated Lennard–Jones potential while the effect of the fragmentation-induced forces was represented by assigning the particles some initial velocities. The energy imparted by the fracturing forces was measured by the kinetic energy due to the assigned initial velocities. For a range of energy input, an effective power-law region was found in $F(m)$. Moreover, the power-law exponent $\alpha$ was found to increase with the energy input.

Motivated by the numerical work of Ching et al. [5], we have performed an experimental study of fragmentation by dropping glass rods onto the ground with a focus on the dependence of the distribution of fragment mass on the energy input. The energy input is conveniently changed by varying the height from which the glass rods are dropped. For lower heights, we find that the differential fragment mass distribution, $n(m)$, is well approximated by one power law. However for higher heights, $n(m)$ has to be represented by two power laws. The power-law exponent for small fragment mass increases and approaches an asymptotic value as the dropping height is increased.

2. The experiment

We study brittle fracture of long glass rods dropped horizontally onto the ground with a focus on the dependence of the distribution of fragment mass on the input energy
that causes fragmentation. Similar fragmentation of glass rods was studied before by Ishii and Matsushita [6]. Potential energy is lost as the glass rod is dropped from a height onto the floor. If all the energy is imparted to the glass rod when it reaches the floor and causes it to break up into fragments, then the energy input will be simply proportional to the height of the drop. We therefore change the energy input by varying the dropping height.

Long glass rods of 1 m long and 2 mm diameter were used. The average mass of the glass rods is 7.43 g with a variation of 1.1%. To ensure that the glass rods are stress free and that consistent results are obtained, all the glass rods have been annealed at 530°C. Following the procedure used by Ishii and Matsushita [6], we placed a glass rod in a sealed stainless-steel tube, which is 1.5 m long with an inner diameter of 25 mm, and then released the tube from the desired height to concrete floor. We have studied seven different heights which range from 1.2 to 23.6 m. For each height, three different runs were performed.

After each drop, the glass fragments were collected from the steel tube and then weighed using an analytical balance. The balance has a resolution of 0.01 mg and a reproducibility of ± 0.05 mg. The analytical balance has been interfaced to a computer such that data are automatically transferred to a computer. In the weighing process, we make sure that we have measured all the fragments with mass equal or larger than a cutoff mass \( m_c \) of 0.4 mg. The remaining fragments were then weighed together and denoted as the residual mass. The sum of the total mass of all the weighed fragments and the residual mass is within less than 1% of the original mass of the glass rod. This serves as a quality check which ensures that all the fragments have been collected. Before doing the next run of experiment, the steel tube was cleaned and rinsed thoroughly to remove traces of glass fragments.

3. Results and discussion

With the glass fragments weighed for the three runs at each height, we calculate both the differential fragment mass distribution \( n(m) \) and the cumulative fragment mass distribution \( F(m) \). \( n(m) \Delta m \) gives the number of fragments whose mass is between \( m \) and \( m + \Delta m \). As the number of fragments decreases when the fragment mass increases, we have used different bin sizes for different mass range in order to get the best statistics.

For the two lowest dropping heights, 1.2 and 2.0 m, \( n(m) \) can be well approximated by one power law

\[
n(m) = A m^{-\beta}
\]

with \( \beta > 1 \). Due to the finiteness of the glass rod, \( n(m) \) only extends up to a finite maximum \( m_L \). The cumulative mass distribution \( F(m) \) is thus related to \( n(m) \) as follows:

\[
F(m) = \int_{m}^{m_L} n(m') \, dm'.
\]
As a result, $F(m)$ is not a simple power law but is given by

$$F(m) = \frac{A}{\beta - 1} \left[ m^{-(\beta - 1)} - m_L^{-(\beta - 1)} \right].$$  

(4)

Nonetheless, $F(m)$ would have an effective power-law region for $m \ll m_L$: $F(m) \sim m^{-\alpha}$ where the exponent $\alpha$ is equal to $\beta - 1$. We show in Fig. 1a the log–log plot of $n(m)$ for $1.2 \text{ m}$. We see that the mass of the fragments covers almost four decades. In Fig. 1b, we plot $F(m)$ for the same dropping height. The solid line is evaluated using (4) with the values of $A$ and $\beta$ obtained by fitting $n(m)$ with (2), which can be seen to fit the data very well. Thus, the ‘curving’ of $F(m)$ for large $m$ in a log–log plot is entirely due to the finite extent of $n(m)$. 

![Fig. 1. (a) Differential fragment mass distribution $n(m)$ for dropping height 1.2 m. The solid line is a fit to a power law. (b) Cumulative fragment mass distribution $F(m)$ for the same dropping height. The solid line is a fit using (4) and the values of $A$ and $\beta$ obtained by fitting $n(m)$ to one power law as shown in (a).](image)
For higher dropping heights, \( n(m) \) is no longer fitted by one power law but has to be represented by two power laws

\[
 n(m) = \begin{cases} 
 A_1 m^{-\beta_1}, & m \leq m_t, \\
 A_2 m^{-\beta_2}, & m > m_t 
\end{cases}
\]

with \( \beta_2 > \beta_1 > 1 \). Consequently, \( F(m) \) has the following form:

\[
 F(m) = \begin{cases} 
 \frac{A_1}{m_t^{(\beta_1 - 1)}} [m^{-\beta_1} - m_t^{-(\beta_1 - 1)}] + \frac{A_2}{m_t^{(\beta_2 - 1)}} [m_t^{-\beta_1} - m_t^{-(\beta_2 - 1)}], & m \leq m_t, \\
 \frac{A_1}{m_t^{(\beta_1 - 1)}} [m^{-\beta_1} - m_t^{-(\beta_2 - 1)}], & m > m_t. 
\end{cases}
\]

Hence, we would observe two effective power-law regions in \( F(m) \), one for \( m \ll m_t \) and the other for \( m_t \ll m \ll m_L \), with exponents \( 1 = \beta_1 - 1 \) and \( 2 = \beta_2 - 1 \), respectively. In Fig. 2, we show \( n(m) \) and \( F(m) \) for the dropping height of 14.8 m. The solid line is evaluated using (6) while the dotted line is evaluated using (4). Clearly \( n(m) \) cannot be approximated by one power law but is better represented by two power laws. We note that this feature of two power-law regions with the exponent larger in the larger mass region is consistent with the results for \( F(m) \) reported in Ref. [6] (see, for example, Figs. 2 and 4 there).

The crossover occurs at \( m_t \), which is of the order of 0.01 g for all the five higher dropping heights studied. If we take the glass rod to be one-dimensional and use the linear mass density to estimate the mass of a fragment with a length of the diameter, we get 0.015 g, which is close to \( m_t \). This indicates that for those fragments with mass \( m \ll m_t \), the fragmentation is three-dimensional. One might be tempted to further interpret the crossover as a crossover from one-dimensional fragmentation of a glass rod to three-dimensional fragmentation of a glass lump [6]. However, it then remains puzzling as to why such a crossover is absent for the two lowest dropping heights. Moreover, together with the finding of earlier studies [2,3] that the exponent \( \alpha \) increases with the effective dimensionality of the object, it seems that such an interpretation should suggest that \( \beta_1 \) larger than \( \beta_2 \), which is not what is observed in the present study.

We compare \( F(m) \) for the seven dropping heights in Fig. 3. To better facilitate the comparison, we plot \( F(m)/F(m_t) \). If we focus on the small mass region, we see that the effective power-law region decreases in extent while the power-law exponent increases as the dropping height increases. These observations agree with those reported in a numerical study using molecular dynamics simulations [5]. Note that \( F(m_t) \) is just the total number of fragments with mass \( m \geq m_t \). This number, denoted as \( N_0 \), is found to increase with the dropping height as expected. For dropping heights lower than 10 m, the rate of increase of \( N_0 \) is about 300/m, while for higher dropping heights, the rate decreases to about 40/m. In Fig. 4, we plot the power-law exponent in \( n(m) \) for the small mass fragments, that is, \( \beta \) or \( \beta_1 \), as a function of the dropping height. The exponent generally increases and approaches an asymptotic value of 1.5 ± 0.05 at the highest heights.
Fig. 2. (a) Differential fragment mass distribution $n(m)$ for dropping height 14.8 m. The solid lines are the fits of $n(m)$ by two power laws. (b) Cumulative fragment mass distribution $F(m)$ for the same dropping height. The solid line is a fit using (6) and the values of $A_{1,2}$ and $\beta_{1,2}$ obtained by fitting $n(m)$ to two power laws as shown in (a).

4. Summary

We have studied brittle fracture of long glass rods dropped onto concrete floor from heights ranging from 1.2 to 23.6 m, with a focus on the dependence of the distribution of fragment mass on the energy input. We find that the distribution of fragment mass does depend on the energy input. This energy dependence is better characterized by studying directly the differential mass distribution $n(m)$. The functional form of $n(m)$ changes from one power law to two power laws as the dropping height or the energy input increases. Moreover, the power-law exponent for fragments with small mass increases and approaches an asymptotic value. Such an energy dependence should be reproduced in any physical model of impact fragmentation. In most, if not all, of the
Fig. 3. Normalized cumulative fragment mass distributions for all the seven dropping heights. The dropping height increases from top to bottom of the curves.

Fig. 4. The power-law exponent in the differential mass distribution for fragments with small mass, \( \beta \) or \( \beta_1 \), as a function of the dropping height \( h \) (m).

earlier work, the cumulative mass distribution \( F(m) \) is studied instead. We have shown that due to the finite extent of \( n(m) \), which is inevitable as we are always studying fragmentation of objects of finite mass, \( F(m) \) is not a simple power law but appear to ‘curve’ downwardly in the large-mass region even when \( n(m) \) is well represented by one power law. The changes of \( n(m) \) as a function of the dropping height imply that when one focuses on the small mass region of \( F(m) \), one would find the power-law region to decrease in extent and the exponent increases as the energy input increases. These changes in \( F(m) \) have indeed been reported in a numerical study of fragmentation using molecular dynamics simulations [5]. However, theoretical understanding is yet to be achieved.
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References