Survey of Spectroscopic Factors for Li – Cr Isotopes

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Abstract

In general, spectroscopic factors found in the literature show great fluctuations as a result of using different optical model potentials and parameters in the reaction theories. It is therefore difficult to pin down the precise nuclear structure information from experiments. We have reanalyzed the angular distributions of the ground-state to ground-state transitions from (d,p) and (p,d) transfer reactions on different Ca isotopes thus obtaining the spectroscopic factors of the valence neutron for $^{40}$Ca, $^{41}$Ca, $^{42}$Ca, $^{43}$Ca, $^{44}$Ca, $^{45}$Ca, $^{46}$Ca, $^{47}$Ca, $^{48}$Ca, $^{49}$Ca isotopes. In our analysis, we used the Johnson-Soper adiabatic and distorted-wave theories and a standard set of parameters including global optical potentials. The parameters were tested previously using the $^{12}$C(d,p)$^{13}$C and $^{13}$C(p,d)$^{12}$C reactions [1]. For the Ca isotopes, the results show good agreement with the predictions of the independent particle model and the shell model. We have extended the method to extract spectroscopic factors for 75 nuclei from Li (Z=3) to Cr (Z=24). The experimental spectroscopic factors for most nuclei agree with the theoretical predictions from the present shell model to within 20%.
Contents

1. Introduction
   1.1 Shell Model
   1.2 Spectroscopic Factor

2. Objective of the project

3. Experimental Spectroscopic Factor
   3.1 Collection of data
   3.2 Calculation of theoretical differential cross sections
   3.3 Extracting spectroscopic factor
   3.4 Results

4. Calculations of Theoretical Spectroscopic Factor
   4.1 Shell Model calculation [Oxbash]
   4.2 Austern’s rule

5. Comparison of Experimental and Theoretical Spectroscopic Factors

6. Endt’s Best Values of Spectroscopic Factors

7. Conclusion

8. Acknowledgement
1. Introduction

1.1 Shell Model

The Shell Model proposed by Mayers and Jensen who shared the Nobel Prize in 1963 has provided successful description of nuclear structure. The shell model, often called the independent particle model, is based upon the motion of nucleon in an averaged field potential provided by the other nucleons. It is analogous to the atomic model that electrons populate shells, and once a shell is filled, the next electron is forced to fill another shell. In shell model, shells exist for both protons and neutrons individually and nuclei are unusually stable when the number of protons or neutrons is equal to the magic numbers: 8, 20, 28, 50, 82 or 126.

Fig 1.1 Shell Model for neutrons in $^{42}$Ca nucleus

1.2 Spectroscopic factor

Theoretical Spectroscopic Factor

Spectroscopic factor is a number which represents how good one can describe the orbit of the valence nucleons and provides nuclear structure information. In stripping reaction, spectroscopic factor is related to the expansion of the wave function for a specific state in the initial nucleus with $A$ nucleons $\Psi_i^A$ in terms of a summation over the complete set of states in the final nucleus with $A+1$ nucleons $\Psi_f^{A+1}$:

$$\Psi_i^A = \sum_f \Phi_f (\vec{r}) \Psi_f^{A+1}$$

where $\Phi_f (\vec{r}) = \langle \Psi_f^{A+1} | \Psi_i^A \rangle$ is the overlap integral

The spectroscopic factor $S$ is defined by the square of the normalization of the overlap integral between the wave function of the valence nucleon in state of target nucleus and that of residual nucleus:

$$S = (\int \Phi_f (\vec{r}) d\tau)^2$$
Experimental Spectroscopic Factor

Experimental spectroscopic factor is defined as the ratio of the experimental differential cross sections to the theoretical differential cross sections:

\[ S = \frac{\frac{d\sigma}{d\Omega}}{\frac{d\sigma}{d\Omega}}_{\text{exp}} \]

where \( \frac{d\sigma}{d\Omega} \) represents differential cross sections

Differential cross sections represent the probability of the particles scattered per unit time within the element of solid angle in the direction with respect to the incident beam.
2. Objective of the project

Spectroscopic factor can be extracted from experimental angular distributions of different single nucleon transfer reactions, such as (p,d), (d,p) and (He,t). As the spectroscopic factor reflects the nuclear structure, it should be independent of reactions and incident energies.

However instead of a constant value independent of energies of the reactions, the experimental spectroscopic factors published in the last 40 years show great fluctuation. Take Carbon isotopes as an example. Figure 3 shows the published spectroscopic factors of $^{12}\text{C}(d, p)^{13}\text{C}$ (closed point) and $^{13}\text{C}(p, d)^{12}\text{C}$ (open point) reactions. It is clear that the published experimental spectroscopic factors fluctuated by nearly a factor of 3. Such fluctuation is the consequence of using different optical model potentials and parameters in extracting experimental spectroscopic factors. Not much useful information of nuclear structure can be obtained based on the inconsistency of spectroscopic factors. Therefore a systematic way should be developed in the interest of better consistency.

Recently it has been shown that with a consistent set of physical inputs published in ref [1], the deduced experimental spectroscopic factors of Carbon isotopes are consistent to within 15% for the incident deuteron energies from 12 to 60 MeV. (Fig 2.2). Such consistency can allow us to apply this technique to a large data set which may provide interesting physics insights.
In this project, the ground state to ground state neutron experimental spectroscopic factors for 79 nuclei ranging from Li to Cr are extracted by analyzing the past measurements of the angular distributions from (d,p) and (p,d) reactions with the same algorithm developed in ref [1]. To gain better understanding of nuclear structure and to verify the current shell model, theoretical spectroscopic factors are calculated by using modern shell model calculation [Oxbash].

3. Experimental Spectroscopic Factor

3.1 Collection of Data

In the present work, we focus on the A(d,p)B and B(p,d)A transfer reactions where the valence neutrons in ground state B are transferred to the ground state A or vice versa. In the past 40 years, the angular distributions of these reactions have been measured extensively. Most of the experimental distributions we used are found in the published literature (mainly from Journal of Nuclear physics and Physical Review) and digitized from the published figures. Only the reactions with incident energies between 10 MeV to 100 MeV are under consideration. It is because the resonance effect and compound nuclear effects are dominant at low incident energies, which are not included in our computer code. On the other hand, different parameterizations may be adopted at high incident energies.

3.2 Calculation of theoretical differential cross sections

Experimental spectroscopic factors are extracted by taking the ratios of the experimental differential cross sections to the theoretical differential cross sections. While
experimental values are obtained from literature, the theoretical one can be calculated by using a modified version of the code TWOFR based on the Distorted-Wave Born Approximation (DWBA) transfer reaction model. In this reaction model, we calculate the transfer differential cross sections within the Johnson-Soper adiabatic approximation to the neutron, proton and target three-body system which includes the effect of break up of deuteron in the field of the target. The transition amplitude of the transfer reaction \( A(a,b)B \) in DWBA model can be written down as:

\[
T = \int \int \frac{d^2 r_a d^2 r_b}{|\mathbf{K}|} \chi^{(-)}(\mathbf{k}_a, \mathbf{r}_a)^* \left( V \Phi_A \right) \chi^{(+)}(\mathbf{k}_b, \mathbf{r}_b)
\]

where \( \chi^{(-)} \) and \( \chi^{(+)} \) are the wave functions describing the elastic scattering of the scattered and incident particles respectively
\( \Phi_B \) and \( \Phi_A \) are the wave functions of the final and initial nuclei respectively
\( \mathbf{K} \) is the momentum
\( \mathbf{r} \) is the relative coordinates
\( V \) is the interaction potential

In addition, we adopted the Chapel Hill global optical model potential set for proton and neutron. All calculations make the local energy approximation (LEA) for finite range effects using the zero-range strength (\( D_0 \)) and range (\( \beta \)) parameters of the Reid Soft core \( ^3S_1-^3D_1 \) neutron-proton interaction. Nonlocality corrections are included in the entrance and exit channel.

Same set of parameters published in ref [1] is used in the reaction model calculations in order to eliminate the inconstancy and uncertainty of extracted spectroscopic factors. The summary of the input parameters are in table 1.

<table>
<thead>
<tr>
<th>Calculation</th>
<th>Adiabatic CH</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proton potential</td>
<td>Chapel-Hill</td>
</tr>
<tr>
<td>Deuteron potential</td>
<td>Adiabatic from CH</td>
</tr>
<tr>
<td>n-binding potential</td>
<td>Woods-Saxon</td>
</tr>
<tr>
<td></td>
<td>( r_a = 1.25, a = 0.65 )</td>
</tr>
<tr>
<td>Hulthen finite range factor</td>
<td>0.7457</td>
</tr>
<tr>
<td>Vertex constant ( D_0^2 )</td>
<td>15006.25</td>
</tr>
<tr>
<td>JLM potential scaling ( \lambda )</td>
<td>N/A</td>
</tr>
<tr>
<td>Non-Locality potentials</td>
<td>p 0.85; n N/A; d 0.54</td>
</tr>
</tbody>
</table>

Tab 1. The summary of physical inputs
3.3 Extracting spectroscopic factor

Spectroscopic factor is extracted by fitting the theory to the experimental data at the first peak of the angular distributions. The backward angles are not used because they are more sensitive to inelastic coupling and high order effect. In figure 3.1, the curve and the points represent the theoretical differential cross sections and the experimental differential cross sections respectively. The open points are the data used to extract spectroscopic factors. The theoretical results are normalized by the spectroscopic factor to give overall fit to the experimental data.

Fig 3.1 The graphs of differential cross sections of transfer reactions for Ca isotopes at different energies

3.4 Results

In this project, we adopted the fixed parameters in DWBA calculations for the entire range of nuclei. We investigated more than 400 angular distributions to extract experimental spectroscopic factors for 79 nuclei form Li to Cr. The preliminary results are shown in Fig 3.2. The exact results will be published in the near future.

Fig 3.2 The experimental spectroscopic factors of different nuclei
4. Calculations of Theoretical Spectroscopic Factor

4.1 Shell Model Calculations [Oxbash]

Oxbash is a set of codes which was first developed by W.D.M. Rae and C.H. Zimmerman in 1976 to carry out shell model calculations. During the last 28 years, this set of codes has been modified for several times. Nowadays, Oxbash is modified as a window PC version and becomes one of the most accurate codes of shell model calculations in the world.

In order to further understand the structure of nuclei, we performed Oxbash to calculate the spectroscopic factors for 59 nuclei. Due to the limitation of computer space, the theoretical spectroscopic factors for heavier nuclei (Z>20) cannot be calculated easily.

4.2 Austern’s rule

Based on the simple independent particle model without considering the interaction between nucleons and core, Austern has derived a simple rule to calculate the theoretical spectroscopic factors for perfect spherical nuclei. For nuclei with n valence neutron, the spectroscopic factor is given by:

\[ S = n \quad \text{when } n \text{ is even} \]

\[ S = 1 - \frac{n-1}{2j+1} \quad \text{when } n \text{ is odd} \]

where j is the quantum number of the orbit

5. Comparison of Experimental and Theoretical Spectroscopic factors

In comparison with the shell model calculation, the experimental spectroscopic factors for most of the nuclei (except deformed Ne, F and Ti isotopes) are in good agreement with the predicted values to within 20%. (shown in Fig 5.1) The good agreement suggests the success of our consistent and systematic method for extracting experimental spectroscopic factors and the success of the present shell model in describing the structure of most of the nuclei ranging in Z from 3-24. There are discrepancies in the theoretical and experimental spectroscopic factors for the deformed nuclei. Further experiments are required in order to understand such difference.
Fig 5.1 The plot of experimental spectroscopic factors against the theoretical values given by Shell Model calculations.

By comparing the experimental spectroscopic factors with the prediction of the simple independent particle model (Austern’s rule), we can observe that most of the experimental values are smaller than those predicted by Austern’s rule. Such discrepancies may arise from the neglect of interaction between nucleons and core in the simple independent particle model. (shown in Fig 5.2)

Fig 5.2 The Plot of experimental spectroscopic factors against the theoretical values given by Austern’s rule

From the figure 5.2, we can find that those points represent Ca isotopes are close to the pink straight line, which implies the simple independent particle model can provide good description to the structure of Ca isotopes. It is true that the Austern’s rule is only appropriate for the perfect spherical nuclei. Therefore it can be concluded that the shape of Ca isotopes is uncommonly spherical. Another plot is given for better illustration. (Fig 5.3)
6. **Endt’s best value of spectroscopic factor**

Analyzing spectroscopic factors with a consistent method is important to the understanding of nuclear structure. In 1977, P.M. Endt compiled a list of the “best” spectroscopic factors for single-nucleon stripping and pick up reaction in the $A = 21 - 44$ region [2]. In his paper, all the spectroscopic factors published were recalculated with the same set of normalization constants. This led to the internal consistency of the spectroscopic factors for different reactions. However his method to remove the normalization uncertainties was greatly based on the communication between authors. On the contrary, our method can provide a more consistent and systematic calculation of spectroscopic factors which can be carried out easily as long as the data of differential cross sections are available.

7. **Conclusion**

A consistent and systematic method is used to extract spectroscopic factors for a wide range of nuclei from $Z = 3$ to 24. Excluding the deformed Ne, F and Ti isotopes, the ground state neutron spectroscopic factors extracted form experimental differential cross sections in the past 40 years are in good agreement with those predicted by present shell model to within 20%. The simple independent particle model calculations provides theoretical spectroscopic factors which are higher than most of the experimental values. Most discrepancies arise from the nucleons or core correlations can be accounted for in the present shell model theory. For the Ca isotopes, the results show good agreement with the predictions of simple independent particle model and the shell model which suggest that the shape of Ca isotopes is nearly spherical.
8. Acknowledge

I would like to thank Professor Betty Tsang for giving me an invaluable opportunity to work on this project and teaching me a lot of knowledge and research techniques. I would also like to thank Professor Bill Lynch and Doctor X.D. Liu for guiding me through my work. In addition, I appreciate the Department of Physics of the Chinese University of Hong Kong and the organizer of SURE program for sending me to the NSCL of Michigan State University and providing financial support.

Reference


[3] Alex Brown, Some Notes on Spectroscopic Factors