Zero Magnetic Field Environment for EDM Measurement

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Abstract
Neutron electric dipole moment measurement (EDM) requires precise control of the
electric and magnetic field in the test cell. External magnetic field should be
shielded out completely. As the experiment will be conducted in cryogenic
environment (~ 4K), conventional μ-metal shield cannot work in low temperature.
Shielding property of another magnetic material, Metglas, was study in this project.
Zero magnetic field was tried to fabricate with Metglas shields and a square coil pair.
# Table of Content

I. Introduction \hspace{3cm} P. 2

II. Overview of the project \hspace{3cm} P. 3

III. Room Temperature Shielding Measurement \hspace{3cm} P. 4
   Transverse Shielding Factor \hspace{3cm} P. 10
   Axial Shielding Factor \hspace{3cm} P. 13

IV. Fabricating Zero Field Region \hspace{3cm} P. 19

V. Cryogenic System \hspace{3cm} P. 26

VI. Conclusion \hspace{3cm} P. 30

VII. Acknowledgement \hspace{3cm} P. 31

Appendix A --- Error Estimation \hspace{3cm} P. 32

Appendix B --- Explicit form of Square coil field \hspace{3cm} P. 33

Reference \hspace{3cm} P. 35
I. Introduction

Neutron Electric Dipole Moment measure

In the Neutron Electric Dipole Moment (EDM) Experiment at the Los Alamos National Laboratory [1], the primary method is to study the precession frequency of the ultra-cold neutrons (UCN). Neutrons and $^3$He were prepared with aligned spin in a bath of superfluid $^4$He at about 300 mK. Since neutron and $^3$He nuclei have finite magnetic dipole moment, both neutrons and $^3$He precess with its own frequency in a plane perpendicular to a static magnetic field, $B_0$. The EDM measurement can be taken by applying a static electric field, $E_0$, parallel and anti-parallel to $B_0$. The electric field can change the precession frequency of the neutron in proportion to neutron EDM, $d_n$, but there is no effect on $^3$He. The neutron EDM can be measured by comparing the difference between the precession frequency of neutron and $^3$He under both $B_0$ and $\pm E_0$.

The precession frequency of neutron is:

$$\nu_n = -(2\mu_n B_0 \pm 2d_n E_0) / h = \nu_0 \pm \Delta\nu$$  \[I.1\]

where $\mu_n$ is the magnetic dipole moment of neutron ($\mu_n < 0$)

$h$ is the Planck constant

The frequency without E-field is

$$\nu_0 = -2\mu_n B_0 / h$$  \[I.2\]

The precession frequency due to the electric field is:

$$\Delta\nu = 2d_n E_0 / h$$  \[I.3\]

The current accepted value of $|\mu_n|$ is $9.662363974 \times 10^{-27}$ J/T and the value of $d_n$ is about $10^{-25} \text{e} \cdot \text{cm}$. To make $\Delta\nu$ large enough to observe relative to $\nu_0$, either a large E-field or a small B-field is required. The static field $B_0$ is chosen to be 1 mGauss. The precession of the magnetic dipole moment of the neutrons and $^3$He nuclei is about 3 Hz. The precession due to $E_0$ is

$$\Delta\nu = 1.450781748 \times 10^6 \text{Hz} = 4.975742857 \times 10^{-7} |\nu_0|.$$  

Therefore a precise measurement of precession frequency up to 1$\mu$Hz is required. The systematic error due to the variation of the B-field inside the test cell can be reduced by shielding out all the external B-field, such as the Earth field and B-field from other electronic apparatus. A study of magnetic shield that can be used in cryogenic environment will be important. This project is concentrated on the study of magnetic shields using Metglas 2705M amorphous ribbons provided by Honeywell International Metglas Solution.
Metglas 2705M Magnetic Shield

Metglas 2705M is a magnetic material, which is a cobalt base amorphous alloy (75-85% Cobalt; 1-5% Boron, Iron, Molybdenum, Nickel; 3-7% Silicon). It has an ultra-high permeability ($\mu \sim 290,000$) so it is suitable for making magnetic shield. Metglas is a better than other magnetic materials, such as $\mu$-metal, to make magnetic shield for EDM experiment because the other materials shield cannot work in cryogenic environment, but Metglas shield is supposed to be work.

II. Overview of this Project

The main objective of this project is to construct a zero field region under cryogenic environment using Metglas shield. Although there is only one main objective, this project consists of 4 components. They are

1) Room Temperature Shielding measurement,
2) Fabricating Zero Field Region,
3) Design of Cryogenic system, and
4) Cryogenic Shielding Measurement

1) Room Temperature Shielding Measurement

In order to study the shielding effect of Metglas, a series of room temperature shielding factor measurements of different shields was done. The shields have different geometric configurations. It helps us to determine what geometry has a better shielding effect. In addition, the shielding factor of a $\mu$-metal was also measured so that a comparison can be made between $\mu$-metal and Metglas.

2) Fabricating Zero Field Region

It’s well known that the Earth magnetic field is about 0.5 Gauss. This field can make the Metglas become magnetized. Once the shield is magnetized, an undesired field will be produced by the shield itself. A technique of demagnetization by applying an alternating B-field through the shield can be used, but it requires an environment with B-field lower than 0.01 Gauss. So, a pair of square coil was constructed to compensate the Earth field.

3) Design of Cryogenic System

This part discusses the design of the cryogenic system that is used for conducting the low temperature shielding measurement.

4) Cryogenic Shielding Measurement
Studying of the shielding effect of Metglas in low temperature (~4K) and construction of a zero field region is the main purpose of this project. But since the fluxgate magnetometer has not arrived yet, this part will be followed up later.

### III. Room Temperature Shielding Measurement

**Physics Properties of Metglas**

Metglas is an amorphous cobalt-based alloy. It is suitable for making flexible electromagnetic shielding, magnetic sensors and high frequency cores. Table III.1 shows the physics properties of Metglas 2705M.

<table>
<thead>
<tr>
<th>Physical Properties</th>
<th>Magnetic Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density (g/cc)</td>
<td>Saturation Induction (T)</td>
</tr>
<tr>
<td>Vicker’s Hardness (50g load)</td>
<td>7.80</td>
</tr>
<tr>
<td>Tensil Strength (GPa)</td>
<td>0.77</td>
</tr>
<tr>
<td>Elastic Modulus (GPa)</td>
<td>900</td>
</tr>
<tr>
<td>Lamination Factor (%)</td>
<td>Maximum DC Permeability (µ)</td>
</tr>
<tr>
<td>Thermal Expansion (ppm/°C)</td>
<td>1-2</td>
</tr>
<tr>
<td>Crystallization Temperature (°C)</td>
<td>100-110</td>
</tr>
<tr>
<td>Continuous Service Temp. (°C)</td>
<td>600,000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Annealed</th>
<th>As Cast</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vicker’s Hardness (50g load)</td>
<td>900</td>
<td>600,000</td>
</tr>
<tr>
<td>Tensil Strength (GPa)</td>
<td>1-2</td>
<td>290,000</td>
</tr>
<tr>
<td>Elastic Modulus (GPa)</td>
<td>100-110</td>
<td></td>
</tr>
<tr>
<td>Lamination Factor (%)</td>
<td>&gt;75</td>
<td></td>
</tr>
<tr>
<td>Thermal Expansion (ppm/°C)</td>
<td>12.1</td>
<td></td>
</tr>
<tr>
<td>Crystallization Temperature (°C)</td>
<td>520</td>
<td></td>
</tr>
<tr>
<td>Continuous Service Temp. (°C)</td>
<td>90</td>
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</table>

<table>
<thead>
<tr>
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<th>Curie Temp. (°C)</th>
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<tr>
<td>Crystallization Temperature (°C)</td>
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</table>

<table>
<thead>
<tr>
<th></th>
<th>Electrical Resistivity (µ-Ω-cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermal Expansion (ppm/°C)</td>
<td>136</td>
</tr>
<tr>
<td>Continuous Service Temp. (°C)</td>
<td>90</td>
</tr>
</tbody>
</table>

Table III.1 Physical properties claimed by manufacturer [2]

**Design of Metglas magnetic shields**

Total seven magnetic shields with different geometry were made. All shields were made of Metgals ribbons and aluminum cylinder (Figure. III.1). The ribbon has the dimension: width = 2 inches (50.8 mm) and thickness = 1 mil (0.00254mm).

![Figure III.1](image)

The shield in the picture is Sample A. The purple coil winding around the shield is used for demagnetizing the shield.

Sample A, B, C, D, E and F were made of an aluminum (Al) cylinder covered with
Metglas ribbons. The Al cylinder was used as a physical support for the Metglas because Al is not a magnetic material. The ribbons were winded to a roll shape (say round structure) and placed around on the outer surface of the Al cylinder as shown in Figure III.2.

![Figure III.2 Round Structure Shield](image)

Sample G was made in a slightly different way. It is also a cylindrical shield. Geometrically, it is identical to Sample F. For Sample G, the ribbons were first cut to be rectangular tapes, which has same length as the cylinder. Then they were stuck along the axis of cylinder (say axial structure) as shown in Figure III.3.

![Figure III.3 Axial Structure Shield](image)

The main difference between those 2 structures of Metglas ribbon is the slits between adjacent Metglas ribbon align in perpendicular direction. The slits in Sample A, B, C, D, E and F are around the shields’ surface perpendicular to the axis of cylinder, while that of Sample G is along the axis.

Table III.2 shows the geometry of all the Metglas shielding samples.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Length</th>
<th>Diameter</th>
<th>Metglas Thickness</th>
<th>No. of Metglas layer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample A</td>
<td>330.2</td>
<td>44.18</td>
<td>1.27</td>
<td>50</td>
</tr>
<tr>
<td>Sample B</td>
<td>50.8</td>
<td>44.18</td>
<td>0.25</td>
<td>10</td>
</tr>
<tr>
<td>Sample C</td>
<td>101.6</td>
<td>44.18</td>
<td>0.25</td>
<td>10</td>
</tr>
<tr>
<td>Sample D</td>
<td>152.4</td>
<td>38.1</td>
<td>0.13</td>
<td>5</td>
</tr>
<tr>
<td>Sample E</td>
<td>152.4</td>
<td>44.18</td>
<td>0.13</td>
<td>5</td>
</tr>
<tr>
<td>Sample F</td>
<td>152.4</td>
<td>63.5</td>
<td>0.13</td>
<td>5</td>
</tr>
</tbody>
</table>
Magnetic Shielding Theory

High permeability materials allow more magnetic flux passing through. It collects flux from the space nearby. Figure III.4 shows the schematic shielding mechanism of a cylindrical shield.

The region inside the shield has a low flux density. It was suggested that the maximum flux density within the shield for transverse case is

$$ H_{mT} = \frac{H_0 D}{t} $$  \hspace{1cm} [III.1]

, for axial case is

$$ H_{mA} = \frac{H_o L}{Kt} \left(1 + \frac{\alpha L}{D}\right) $$  \hspace{1cm} [III.2]

where $\alpha = 0.85 \pm 0.03$

$$ K = \left(1 + \frac{1}{4\alpha^2}\right) \beta - \frac{1}{a} + 2\alpha \left[\ln\left(a + \left(1 + a^2\right)^{1/2}\right) - 2\left(1 + \frac{1}{a^2}\right)^{1/2} - \frac{1}{2}\right] $$  \hspace{1cm} [III.3]

$a = L / R$
$L$ = length of the shield
$R$ = radius of the shield
$D$ = diameter of the shield
$t$ = thickness of the Metglas layer
$H_0$= external filed without shield

The theory applied here is mainly following the review of conventional magnetic shielding by T J Sumner, J M Pendlebury and K F Smith [3], and the idea of E
The static shielding factor is defined by the ratio of the external field $H_0$ to the field within the shielded region $H_s$. The transverse shielding factor is

$$S^T = 1 + \frac{\mu_t}{D}$$  \[\text{III.4}\]

and the axial shielding factor is more complicated,

$$S^A = \left( S_a^{-1} + S_{op}^{-1} \right)^{-1}$$  \[\text{III.5}\]

where

$$S_a = 1 + 4NS^T$$  \[\text{III.6}\]

$$N = \frac{1}{p^2 - 1} \left( \frac{p}{\sqrt{p^2 - 1}} \ln \left( \frac{p + \sqrt{p^2 - 1}}{p - \sqrt{p^2 - 1}} \right) - 1 \right)$$  \[\text{III.7}\]

$$S_{op} \sim \frac{1}{2.6\sqrt{L/D}} e^{k_L/D}$$  \[\text{III.8}\]

$$k_L = 2.405$$

Since there is no exact formulae for finite length cylindrical shielding factor, the formulae above based on an approximate method with length to radius ration $>1$. Note that the both transverse and axial shielding factor depends only on the geometry and the permeability of the shield.

**Experiment on Shielding Factor**

The experiment setup is shown in Figure III.5. A uniform static external field was provided by the Helmholtz coil. A 3-axis probe (connecting to FW Bell 9950 Gaussmeter) was mounted at the center of the coil.
The external field was measured before putting the shield around the probe. Then the shield was put into the field and the first measurement of shielded field was taken. The shield would be magnetized by the external field. There would be a field due to the magnetization of shield. Therefore what we measured will include the effect of magnetization. To reduce the error of it, after the first measurement, the shield was rotated 180° (flipping the 2 ends) and measured the field again.

Since the field inside the shielded region after magnetization is

\[ H_1 = H_s + M \]  

where \( M \) is the field due to magnetization.

After rotation, the direction of \( H_s \) will not change, but that of \( M \) will also rotate 180°. So the field inside the shield becomes

\[ H_2 = H_s - M \]  

By adding up the reading of \( H_1 \) and \( H_2 \) then dividing it by 2, the effect of magnetization can be eliminated. And the result is exactly \( H_s \). The error estimation is discussed in Appendix A.

Experiment Result

Overview of the shielding factors

Transverse shielding measurement was taken on Sample A, B, C, F and G. Axial shielding measurement was taken on Sample A, B, C, E, F and G. Sample D was not measured. In order to compare the shielding effect of Metglas shields with that of other shield, \( \mu \)-metal shield was chosen. It is because \( m \)-metal has a high permeability, \( \sim 10^5 \), which is similar to the permeability of Metglas.

The \( \mu \)-metal shield used here has length, diameter and thickness equal to 152.4mm, 58.5 mm and 1 mm respectively. Its geometry is similar to Sample E, F and G, except the thickness of \( \mu \)-metal which is about 10 times thicker than the other 3 Metglas samples. The comparison of transverse shielding factor is still easy to be done because the shielding factor has a linear relation to the thickness.

There exist a saturation induction of the shield. Therefore the shield cannot collect addition magnetic flux once saturation induction is reached. The saturation induction for Metglas is 7700 Gauss. The external field that makes the shield to be saturated can be calculated by putting \( H_m = 7700 \) Gauss. Table III.3 shows the magnitude of the external field that makes the different Metglas shields saturated.
Table III.3  Magnitude of external field causing saturation induction for both transverse and axial case. The values of \( \mu \)-metal was not calculated because the saturation induction of \( \mu \)-metal was unknown.

Generally, the shielding factor is constant for all \( H_0 \). But the shielding factor will start to drop down when \( H_0 \sim H_{s\text{at}}^T \) and \( H_{s\text{at}}^A \) for transverse case and axial case respectively. When \( H_0 > H_{s\text{at}}^T \) and \( H_{s\text{at}}^A \), the shielding factor will become another constant again.

The graphs below show the shielding factors as a function of \( H_0 \). Since some of the samples have a large shielding factor, the shielded field is too small to be measured accurately with our device. In addition, linear increasing curve appears in the region of inaccurately measurement. It’s because the readings of \( H_s \) keep in a constantly low value, such that the shielding factor will increase linearly as \( H_0 \) increase. However, we can calculate the lower limit of the shielding factor from the estimated uncertainty of \( H_s \) (~±0.004 Gauss). The calculation of lower limit of the shielding factor is:

\[
\text{Maximum Shielded Field} = \text{max. } H_s < H_s + \text{Uncertainty of } H_s
\]

\[
\text{Shielding Factor} = S = \frac{H_0}{H_s}
\]

\[
S > \frac{H_0}{\text{max. } H_s}
\]

\[
\text{Minimum Shielding Factor} = \text{min. } S = \frac{H_0}{\text{max. } H_s}
\]
Figure III.6  Lower limit of transverse shielding factor of Sample A against $H_0$

The shielding factor of Sample A is large. Although the external field is 60.8 ± 0.005 Oe, the shielded field is only 0.002 ± 0.004 Oe. Our coil cannot provide a larger external field, so the range of $H_0$ is limited. From this result, the minimum value of shielding factor was estimated to be 10998.

Figure III.7  Transverse shielding factor of Sample B against $H_0$
This graph shows the ideal relation between shielding factor and external field. At low external field, the shielding factor is nearly constant. It then starts to drop down when the shield starts to be saturated. The error of the data from Sample B is small, less than 10%. The maximum shielding factor is 89.78 ± 2.56. It is smaller than the theoretical value, 1642. It is because the length of sample B is not long enough. As a result, there may be some flux bending into the shielded region easily from two opened ends of the shield. It makes the field inside the shield larger. The length to radius ratio of this shield is only 2.3, while the shielding have optimum effect when the length to radius ratio is 5.5 [3].

Sample C

![Graph showing the relationship between shielding factor and external field](image)

**Figure III.8** Transverse shielding factor of Sample C against $H_0$

The data of shielding factor at low field region have large error so the minimum shielding factors were calculated. Before saturation, there are 2 data, 3778 and 3601 with error ±70.7% and ±58.9% respectively, and they are consistent to the minimum shielding factor from previous data points. The minimum shielding factor was estimated to be 2700.
Sample F

The points in this region show the lower limit of the shielding factors since the absolute % error of those points is larger than 100%.

Figure III.9 Transverse shielding factor of Sample F against $H_0$
For the same reason to the case in Sample C, the lower limits of shielding factors were calculated for small $H_0$. The minimum shielding factor was estimated to be about 1500.

Sample G

Figure III.10 Transverse shielding factor of Sample F against $H_0$
Although the geometry and the material used of ample G is same as that of Sample F, the result of transverse shielding factor is totally different. The maximum shielding
factor read from the graph is $18.40 \pm 0.43$. It is about 100 times smaller than the shielding factor of Sample F.

**µ-Metal Shield**

![Graph showing transverse shielding factor of µ-metal against $H_0$](image)

**Figure III.11 Transverse shielding factor of µ-metal against $H_0$**

The shielding factor was estimated to be $1471 \pm 359$. Using permeability of µ-metal $= 10^5$ (approximation up to one order of magnitude), the theoretic transverse shielding factor is $1710$. The experiment result is consistent to the theoretical value.

*Axial Shielding Factors*

The experiment results of Sample A, C, E and F not agree with the theoretical value. It was suspected to be the effect of the slits of Metglas structure. We will come up to a discussion of this effect after presenting the results.
Sample A

Figure III.12  Axial shielding factor of Sample A against $H_0$

Sample B

Figure III.13  Axial shielding factor of Sample B against $H_0$
Sample C

![Graph showing axial shielding factor of Sample C against $H_0$.](image)

**Figure III.14** Axial shielding factor of Sample C against $H_0$

Sample E

![Graph showing axial shielding factor of Sample E against $H_0$.](image)

**Figure III.15** Axial shielding factor of Sample E against $H_0$
Sample F

Figure III.16  Axial shielding factor of Sample F against $H_0$

Sample G

Figure III.17  Axial shielding factor of Sample G against $H_0$

$\mu$-Metal
Summary of axial shielding factors
The axial shielding factors of Sample A, C, E and F do not agree with the theory. On the other hand, the axial shielding factors of Sample B, G and μ-metal have a better agreement with the theory.

The reason of disagreement of Sample A, C, E and F with the theory may be the effect of the slit between Metglas rolls. The slits cause the spatial discontinuous of the permeability. The permeability changes from sharply at the slits (from Metglas to air). The maximum flux density in the Metglas is large, but when there is slit, the flux density drops down. So when the flux is passing perpendicular to the slits, some of the flux going out from the Metglas to air should be repelled away (Figure III. 17). It then leads flux leakage. The outgoing flux will enter the shielded region. Therefore the shielding factor was reduced.
Figure III.19  The schematic figure of how the slits between Metglas affect the shielding factor. The arrows represent the flux passing through a slit. In the Metglas region more flux is allowed to pass. But in the slit (air), less flux is allowed to pass, so some of them will bend away from the shield.

Since Sample A, C, E and F are in around structure, when the flux is going along the axis of the shield, the flux crosses the slits perpendicularly. Then the flux leakage occurs. The reason why Sample B did not have such effect may be it has only one slit, so the effect is not significant.

This also can explain why the transverse shielding factor of Sample G is 100 times smaller than that of Sample F provided that they have the same geometry. For transverse case, the flux going from the surface of the shielding, they will only cross with the slit that parallel to the axis of shield. The flux leakage only occurs in the shield with axial structure. Therefore Sample G has a smaller transverse shielding factor than that of Sample F.

For the μ-metal shield, the permeability of it is unknown. But the typical value of μ-metal permeability is $10^5$. The theoretical shielding factors of μ-metal are only true up to the order of magnitude. However, both transverse and axial shielding factor of μ-metal is consistent to the theoretical value. It was not surprising because the μ-metal shield is an as cast product, no slits present on the shield.

Experiment Result Summary and Comparison with Theory
Table III.4 shows the summary of the experiment result and the theoretical value of the shielding factors of each sample.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Transverse Shielding Factor</th>
<th>Axial Shielding Factor</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Experiment</td>
<td>Theory</td>
</tr>
<tr>
<td>A</td>
<td>&gt; 10998</td>
<td>8337.0</td>
</tr>
<tr>
<td>B</td>
<td>89.78 ± 2.56</td>
<td>1642.0</td>
</tr>
<tr>
<td>C</td>
<td>&gt; 2700</td>
<td>1642.0</td>
</tr>
<tr>
<td></td>
<td>E</td>
<td>F</td>
</tr>
<tr>
<td>---</td>
<td>-----</td>
<td>---------</td>
</tr>
<tr>
<td></td>
<td>Not measured</td>
<td>833.6</td>
</tr>
<tr>
<td></td>
<td>&gt; 1500</td>
<td>580.0</td>
</tr>
<tr>
<td></td>
<td>18.40 ± 0.43</td>
<td>580.0</td>
</tr>
<tr>
<td></td>
<td>1471 ± 359</td>
<td>1710.0</td>
</tr>
</tbody>
</table>

Table III.4 Summary of transverse and axial shielding factor. The permeability used for calculating the theoretical shielding factor of Metglas and \( \mu \)-metal are 290,000 and 100,000 respectively.

The transverse shielding factors of Sample A, C and F are larger than the theoretical value. For Sample B, the length is too short. For Sample G, the slit effect reduces the shielding factor. So there is systematic error in Sample B and G.

For axial shielding factors, Sample G and \( \mu \)-metal have a shielding factor larger than the theoretical value. The slit effect occurs in the other samples, so the shielding factor of them is smaller than the theoretical value.

It was concluded from the result of transverse shielding factor that the permeability of Metglas may be larger than 290,000. The minimum value of permeability of Metglas was calculated from the results of transverse shielding factor of Sample A, C and F. The results are 382557, 476967 and 749500 calculated from Sample A, C and F respectively. The permeability cannot be obtained from the result of axial case. Negative values were calculated from result of Sample G and \( \mu \)-metal, which were not affected by slit and length effect and more reliable.

### IV. Fabricating Zero Field Region

Metglas can be easily magnetized in low magnetic field. A 0.05 Gauss field can completely magnetized the Metglas. As a result, a mechanism of demagnetization should be applied to the Metglas shield. The demagnetization method we used is by applying an ac magnetic field passing through the Metglas with decreasing magnitude. The ac magnetic field is produced by the coil winding around the shield. The demagnetization coil was only built on Sample A.

In order to compensate the Earth field, a pair of square coil was designed and constructed. The square can produce a uniform B-field pointing to one specific direction. So it can be adjust to a manner that the vector sum of the Earth field and the coil field is zero.
**Calculation of Magnetic field of a Square coil pair**

**Objective:**
To calculate the B-field as a function of spatial coordinates of a square coil pair with length $2R$ and coil separated by $L$.

![Square Coil Pair](image)

**Method:**
Add up the field provided by every single straight segment of current wire. Since the field of a single straight segment of wire is well known, transformation of spatial coordinates and direction of field is the only need.

**Calculations:**
Unit of magnetic field is Tesla.
Unit of length is meter.

B-Field of single segment wire with the origin placing at the middle of the wire:

![B-Field of single segment wire](image)

The magnitude of the B-field a distance $s$ from a straight wire with constant current $i$ is given by
To calculate one side of a square coil in 3D space, transformation of \( B_{\text{segment}} \) can be used.

![B-field due to segment of wire in 3D space](image)

The vector \( B_{\text{segment}}'(x, y, z) \) represents the vector field at the point \((x, y, z)\) in Figure IV.2. The perpendicular distance from the wire to \((x, y, z)\) is

\[
s = \sqrt{y^2 + z^2}
\]

The field due to a wire segment with current \( i \) at \((x, y, z)\) is

\[
B_{\text{segment}}^x = 0
\]

\[
B_{\text{segment}}^y = -B_{\text{segment}}(x, s) \sin \theta = -B_{\text{segment}}(x, \sqrt{y^2 + z^2}) \frac{z}{\sqrt{y^2 + z^2}} \tag{IV.2}
\]

\[
B_{\text{segment}}^z = B_{\text{segment}}(x, s) \cos \theta = B_{\text{segment}}(x, \sqrt{y^2 + z^2}) \frac{y}{\sqrt{y^2 + z^2}}
\]

, the superscript \( x, y \) and \( z \) represent the \( x, y, z \)-component of vector \( B_{\text{segment}}' \) respectively.

Then we transform the origin to the center of the bottom square coil and the new coordinate is noted by \( x', y', z' \).

![Diagram of wire configuration](image)
Figure IV.3  The dot represent the center of the square coil, i.e. the new origin. x-axis is pointing down from the mid-point of each wire. z-axis is pointing out from the paper. \((x, y_1, z)\) is the coordinate of wire 1. \((x, y_2, z)\) is the coordinate of wire 2.

In Figure IV.3, the transformation for wire 1 is

\[ \begin{align*}
x' &= x \\
y' &= y_1 - R \\
z' &= z
\end{align*} \]

for wire 2 is

\[ \begin{align*}
x' &= x \\
y' &= y_2 + R \\
z' &= z
\end{align*} \]

The B-field due to the pair of current wire in Figure IV.3 at new coordinate \((x', y', z')\) is

\[ \vec{B}_{\text{pair}} (x', y', z') = \vec{B}_{\text{segment}} '(x', y'+R, z') - \vec{B}_{\text{segment}} '(x', y'-R, z') \]  \[ \text{[IV.3]} \]

, the minus sign appears because the current of wire 2 is running opposite direction to wire 1.

B-field at \((x', y', z')\) due to another pair of wire can be obtained from \(\vec{B}_{\text{pair}}\) by transformation of coordinate and rotation of the direction of B-field.

Figure IV.4  The figure shows old coordinate and the new coordinate of a pair of current wire.

The transformation from \((x', y', z')\) to \((x'', y'', z'')\) is exactly a anti-clockwise rotation by 90°, which is

\[ \begin{align*}
x'' &= -y' \\
y'' &= x'
\end{align*} \]
In addition, the direction of B-field should be rotated 90° in anti-clockwise direction. The rotation is done by multiplying a matrix,

\[
\begin{pmatrix}
0 & -1 & 0 \\
1 & 0 & 0 \\
0 & 0 & 1
\end{pmatrix}
\]

The B-field due to the rotated wire pair is

\[
\vec{B}_{\text{pair}}'(x'', y'', z'') = \begin{pmatrix}
0 & -1 & 0 \\
1 & 0 & 0 \\
0 & 0 & 1
\end{pmatrix} \cdot \vec{B}_{\text{pair}}(y'', -x'', z'')
\]

By addition up the B-field due to both wire pair, we have the B-field due to a square coil, that is

\[
\vec{B}_{\text{coil}}(x'', y'', z'') = \vec{B}_{\text{pair}}'(x'', y'', z'') + \vec{B}_{\text{pair}}'(x'', y'', z'')
\]

The origin is now at the center of a square coil. It has a better symmetry if the origin was transformed to the center of the square coil pair. Therefore, the origin of the bottom square coil should be transformed upward with distant \(L/2\). On the other hand, the origin of the top square coil should be transformed downward with distant \(L/2\). The total B-field due to the whole square coil pair is

\[
\vec{B}(x, y, z) = \vec{B}_{\text{coil}}(x, y, L/2 + z) + \vec{B}_{\text{coil}}(x, y, L/2 - z)
\]

For conventional purpose, we use \((x, y, z)\) to represent the coordinate with origin at the center of the square coil pair. The explicit form of \(\vec{B}(x, y, z)\) is shown in Appendix B.

*Magnetic field measurement of square coil pair*

A square coil pair with \(L = 30\text{cm}, R = 50\text{cm} and number of turns of coil = 30\) was built and 1 Am current was applied to the coil.

The gaussmeter we used was FW Bell 9950 gaussmeter with a 3-axis probe. To measure the net effect of the coil, the earth field was first measured. Then the 1 Am was applied to the coil and measure the field again. By subtracting the Earth field from the field with the field turned on for each component, the net field due to the coil can be obtained.
In order to make sure the probe’s z-axis to be parallel to the z-axis of the square coil, for \( z = 0 \), the probe should be adjust such that the reading of x and y components with and without applied field is the same. It’s because there is no x and y component for \( z = 0 \).

**Measurement Result and Comparison with Theory**

The magnitude and the variation of the B-field were important. Table IV.1 shows the z component and the magnitude of the field, and also the comparison between the experiment result and the theoretical value.

<table>
<thead>
<tr>
<th>Position (x,y,z)</th>
<th>Experiment B-field</th>
<th>Theory B-field</th>
<th>Compare Exp. result vs Theory (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( H_x )</td>
<td>(</td>
<td>H</td>
</tr>
<tr>
<td>4.75</td>
<td>0</td>
<td>0</td>
<td>617</td>
</tr>
<tr>
<td>2.5</td>
<td>0</td>
<td>0</td>
<td>608</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>605</td>
</tr>
<tr>
<td>-2.5</td>
<td>0</td>
<td>0</td>
<td>609</td>
</tr>
<tr>
<td>-5</td>
<td>0</td>
<td>0</td>
<td>621</td>
</tr>
<tr>
<td>4.75</td>
<td>2.5</td>
<td>0</td>
<td>618</td>
</tr>
<tr>
<td>2.5</td>
<td>2.5</td>
<td>0</td>
<td>608</td>
</tr>
<tr>
<td>0</td>
<td>2.5</td>
<td>0</td>
<td>607</td>
</tr>
<tr>
<td>-2.5</td>
<td>2.5</td>
<td>0</td>
<td>609</td>
</tr>
<tr>
<td>-5</td>
<td>2.5</td>
<td>0</td>
<td>622</td>
</tr>
<tr>
<td>4.75</td>
<td>5.25</td>
<td>0</td>
<td>627</td>
</tr>
<tr>
<td>2.5</td>
<td>5.25</td>
<td>0</td>
<td>619</td>
</tr>
<tr>
<td>0</td>
<td>5.25</td>
<td>0</td>
<td>618</td>
</tr>
<tr>
<td>-2.5</td>
<td>5.25</td>
<td>0</td>
<td>621</td>
</tr>
<tr>
<td>-5</td>
<td>5.25</td>
<td>0</td>
<td>634</td>
</tr>
<tr>
<td>4.75</td>
<td>-2.5</td>
<td>0</td>
<td>644</td>
</tr>
<tr>
<td>2.5</td>
<td>-2.5</td>
<td>0</td>
<td>608</td>
</tr>
<tr>
<td>0</td>
<td>-2.5</td>
<td>0</td>
<td>610</td>
</tr>
<tr>
<td>-2.5</td>
<td>-2.5</td>
<td>0</td>
<td>617</td>
</tr>
<tr>
<td>-5</td>
<td>-2.5</td>
<td>0</td>
<td>629</td>
</tr>
</tbody>
</table>
Table IV.1  The unit of magnetic field is mGauss, and the unit of length is inch.
The error of $H_z$ and $|H|$ is ±5 mGauss.

$$
\Delta H_z = |H_z(\text{experiment}) - H_z(\text{theory})| \quad \Delta |H| = |H(\text{experiment}) - |H(\text{theory})| |
$$

From Table IV.1, the largest difference between the experiment result and the theoretical value is 2.8%.

Table IV.2 shows the magnitude of B-field of the square coil for $z = -6$ inches.

<table>
<thead>
<tr>
<th>Position (x,y,z)</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>y</td>
<td>z</td>
<td>$</td>
<td>H</td>
<td>$</td>
<td>$\Delta H/H(0,0,0)$ (%)</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>-6</td>
<td>566</td>
<td>6.41</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>-6</td>
<td>568</td>
<td>6.07</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>-6</td>
<td>568</td>
<td>6.13</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>-6</td>
<td>573</td>
<td>5.29</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.75</td>
<td>0</td>
<td>-6</td>
<td>571</td>
<td>5.60</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>-2.5</td>
<td>-6</td>
<td>570</td>
<td>5.86</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>-2.5</td>
<td>-6</td>
<td>570</td>
<td>5.86</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>-2.5</td>
<td>-6</td>
<td>571</td>
<td>5.56</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>-2.5</td>
<td>-6</td>
<td>576</td>
<td>4.83</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.75</td>
<td>-2.5</td>
<td>-6</td>
<td>577</td>
<td>4.62</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>-5</td>
<td>-6</td>
<td>577</td>
<td>4.64</td>
<td></td>
<td></td>
</tr>
<tr>
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<td>-5</td>
<td>-6</td>
<td>580</td>
<td>4.14</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>-5</td>
<td>-6</td>
<td>580</td>
<td>4.11</td>
<td></td>
<td></td>
</tr>
<tr>
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<td>-5</td>
<td>-6</td>
<td>583</td>
<td>3.58</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.75</td>
<td>-5</td>
<td>-6</td>
<td>586</td>
<td>3.23</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table IV.2  This table shows the percentage difference between the field at $(x,y,z)$ and the field at origin $(0,0,0)$.

From the theory, the square coil pair can produce a cylindrical region, with radius = 5 inches and length = 10 inches, at the center of the coil pair (Figure IV.5) where the magnitude of B-field varies within 5%. According to Table IV.1 and IV.2, we
concluded that the field produced by the square coil has a good agreement with the theoretical result.

![Square Coil Pair](image)

**Figure IV.5** The gray cylindrical region represent the uniform region of the field.

Once the magnitude and direction of the Earth field is known, we can adjust orientation of the square coil and the applied current to compensate the Earth field.

The direction of the Earth field can be measured by using a 3-axis probe. But there is an error due to the probe’s orientation. For example, the z-axis of the probe may not be exactly pointing perpendicular to the surface of the floor or table. Therefore, there may be some unknown angle between the axes relative to the table and the axes of the probe. It increases the difficulty to compensate the Earth field since the information from Table IV.3 may have unknown errors. Try and error method can be used to adjust the orientation of the square coil. Or, two more pairs of square coil can be built and placed perpendicular each other. So, we can produce an arbitrary field in 3D space. The compensation can be achieved easier by adjusting the applied current of each coil pair instead of rotating the coil.

V. **Cryogenic System**

EDM measurement will be conduct in cryogenic environment (~ 4K). Test of shielding factor in cryogenic environment is important. In this section, we discuss the draft design of cryogenic system for shielding measurement.

*Basic Requirement of the Cryogenic System*

There are 2 major requirements for design of cryogenic system.

1. Reducing Heat Intake, and
2. Mounting of Shield and Probe.

*Dewar and Neck*

The cryogenic system consists of 2 parts. One is the dewar, and the other part is the neck. In order to prevent from heat loss, the design should avoid the direct contact
of the cryogen (liquid helium-4) to the room temperature. Figure V.1 shows the design and dimension of the dewar.

![Dewar Diagram](image)

**Figure V.1 Dewar**

The dewar is a cylinder with two layers. The outer layer was made of metal. The upper part of the inner layer was made of thermal insulator, for example G10, and the lower part was made of metal. The cryogen is stored inside the inner layer. The region between two layers is vacuum so it can reduce the heat loss by convection and conduction. In the figure V.1, A is a space for putting a rubber O-ring into it. The O-ring prevents the warm air from entering to the cavity. B is a screw hole. There are totally eight screw holes on the dewar. The screws are used for holding the neck and the dewar together. The neck consists of 11 layers of aluminum. In between each layer, there is a Styrofoam layer. The aluminum layers are used for reducing the heat entering by radiation. Suppose there is only one layer of aluminum, which is in room temperature. The radiation power entering the dewar of the aluminum layer is
\[ P = \frac{1}{2} \sigma \varepsilon A T_{room}^4 \]  \[ [V.1] \]

where \( \sigma \) is the Stefan-Boltzmann constant, \( \sigma = 5.67 \times 10^{-8} \text{ W.m}^{-2}.\text{K}^{-4} \)

\( \varepsilon \) = emissivity of aluminum

\( A \) = Area of the aluminum layer

\( T_{room} \) = room temperature

\( T_{helium} \) = liquid helium-4 temperature

The 1/2 factor presents because only half of the radiation is going into the dewar.

Using \( A = (4 \text{ inches})^2 \pi = 0.032 \text{ m}^2 \), \( T_{room} = 300\text{K} \), \( T_{helium} = 4\text{K} \) and \( \varepsilon = 0.1 \), \( P = 0.74 \text{ W.} \)

Let’s consider the case that 10 additional layers are added. In equilibrium, the net radiation power going down to the dewar between each aluminum layer is the same, otherwise there is energy deposit in the aluminum layer. The net power entering the dewar between layer \( i \) to layer \( i+1 \) is

\[ P_{i,i+1} = \frac{1}{2} \sigma \varepsilon (T_i^4 - T_{i+1}^4) \]  \[ [V.2] \]

where \( T_i \) = Temperature of layer \( i \)

\( T_{i+1} \) = Temperature of layer \( i+1 \)

The net power entering from the top layer (layer 0) is

\[ P_{0,1} = \frac{1}{2} \sigma \varepsilon (T_0^4 - T_1^4) \]

Since the top layer is in room temperature, so \( T_{room} = T_0 \). Let’s look at equation \([V.1]\) again.

\[ P = \frac{1}{2} \sigma \varepsilon T_{room}^4 = \frac{1}{2} \sigma \varepsilon (T_{room}^4 - T_1^4 + T_1^4 - T_2^4 + T_2^4 - T_3^4 + T_3^4 - \ldots - T_{10}^4 + T_{10}^4) \]  \[ [V.3] \]

\[ P = P_{0,1} + P_{1,2} + P_{2,3} + P_{3,4} + P_{4,5} + P_{5,6} + P_{6,7} + P_{7,8} + P_{8,9} + P_{9,10} + \frac{1}{2} \sigma \varepsilon T_{10}^4 \]  \[ [V.4] \]

Equilibrium requires

\[ P_{0,1} = P_{1,2} = P_{2,3} = P_{3,4} = P_{4,5} = P_{5,6} = P_{6,7} = P_{7,8} = P_{8,9} = P_{9,10} \]  \[ [V.5] \]

We have

\[ P - \frac{1}{2} \sigma \varepsilon T_{10}^4 = 10P_{0,1}^4 \]

\[ P_{0,1} = \frac{1}{10} (P - \frac{1}{2} \sigma \varepsilon T_{10}^4) = \frac{1}{10} \frac{1}{2} \sigma \varepsilon (T_{room}^4 - T_{10}^4) \]  \[ [V.6] \]

\( T_{room}^4 \gg T_{10}^4 \) because layer 10 is the nearest layer from the cryogen.
The net power entering the dewar is exactly $P_{0.1}$. Equation [V.7] shows that the net power entering the dewar of a 11-layer neck is 10\% of that of a single layer neck.

The total energy entering the dewar in 1 hour is $\frac{1}{10} \times 0.74 \times 3600 = 266.4 J$. The latent heat of helium-4 is $L = 20.5 \text{ kJ/kg}$ and the density of the helium-4 $\rho = 0.125 \text{ g/cm}^3$. After 1 hour, the height of helium-4 dropped, $h$ is

$$ E = L \rho h A $$

$$ h = \frac{E}{L \rho A} = \frac{266.4}{20500 \times \frac{0.125}{1000} \times 0.032 \text{ cm}^3} $$

$$ h = 0.32 \text{ cm} $$

Note that if there is only 1 layer, the helium drops 3.2 cm per hour.

All the layers can be held together with cryogenic epoxy. On the top layer of the neck, there are eight screw holes at position corresponding to those on the dewar. There should be holes for electronics feed through, shield holder and cryogen transfer inlet.

**Shield and probe holder**

**Shield holder**

The shielding measurement requires the measurement of magnetic field with and without the shield. A movable shield holder is needed. The position of the shield in the dewar should be able to adjust outside the dewar.

**Probe holder**

The probe can be mounted at the bottom of the dewar. It is convenient to make a
base placing at the bottom of the dewar and mount the probe on the base. The base can be a circular disc. Minimizing the volume of the base helps reducing evaporation of liquid helium so the base can be disc with some holes. Figure V.3 shows the schematic design of the shield holder and the base.

![Shield Holder and Base](image)

**VI. Conclusions**

In this study, the room temperature shielding factor of six cylindrical Metglas shields were measured with limited accuracy. The length to radius ratio and the Metglas structure of shield were concluded to have effect on shielding factor. Small length to radius ratio (< 5) reduces the transverse shielding factor. To maximize the shielding factor, the slits between Metglas ribbons should not be perpendicular to the desired shielding direction so that the flux flowing inside the Metglas does not cross any slit. Flux leakage from the slits can be reduced. Shields with different Metglas structure should be used to maximize the shield factor in all directions. For those shield did not have length to radius ratio effect and slit effect, their permeability calculated from the transverse shielding factor (Sample A: 382557, Sample C: 476967 and Sample F: 30...
749500) are higher than the manufacturer’s claim (290,000).

The square coil pair has a good agreement to the theoretical result. However, aligning the probe’s coordinates to be same as the coil coordinates is difficult but a must for precise measurement of the direction of the Earth field. Otherwise, the field compensation cannot be achieved.

Cryogenic shielding measurement can be conduct by putting all the parts of this project together.

**VII. Acknowledgement**
I gratefully acknowledge Professor Bradley Filippone for giving me this opportunity to work in his group and thank for many useful suggestions from Professor Bradley Filippone and Doctor Takeyasu Ito during this project. In addition, I would like to thank the Department of Physics of the Chinese University of Hong Kong for offering me the Summer Undergraduate Research Exchange (SURE) program and the financial support.
Appendix A --- Error Estimation

Shielding factor

Although the resolution of the FW Bell Gaussmeter is 0.001 Gauss, the uncertainty we estimated of the reading was ±0.005 Gauss, because the readings is fluctuating within a range about 0.010 Gauss.

The error of the shielded field after the treatment of canceling the magnetization contribution is

\[
\delta H_s = \sqrt{\left(\frac{\partial H_1}{\partial H_s} \delta H_1\right)^2 + \left(\frac{\partial H_2}{\partial H_s} \delta H_2\right)^2}
\]

\[
\delta H_s = \sqrt{\left(\frac{1}{2} \delta H_1\right)^2 + \left(\frac{1}{2} \delta H_2\right)^2}
\]

\[
\delta H_s = \frac{1}{2} \sqrt{2 \delta H}
\]

Note that: \( H_s = 1/2 \ (H_1 + H_2) \)

\( \delta H_1 = \delta H_2 = \delta H = 0.005 \) Gauss

So, \( \delta H_s = 0.004 \) Gauss

The error of the shielding factor is

\[
\delta S = \sqrt{\left(\frac{\partial S}{\partial H_0} \delta H_0\right)^2 + \left(\frac{\partial S}{\partial H_s} \delta H_s\right)^2}
\]

\[
\delta S = \sqrt{\left(\frac{1}{H_s} \delta H_0\right)^2 + \left(\frac{H_0}{H_s^2} \delta H_s\right)^2}
\]

note that: \( S = \frac{H_0}{H_s} \)

\( \delta H_0 = \delta H_s = 0.005 \) Gauss
Appendix B --- Explicit Form of the Square Coil Field

The expression of the field is calculated by using Mathematica 4.0
Unit of magnetic field is Tesla.  Unit of length is meter.

The x, y and z component of the magnetic field is:

\[ B_x (x, y, z) = \]
\[
\frac{1}{4\pi} \left( \frac{(x_a^2 + y^2)(y_a^2 + z^2)}{(x_a^2 + y^2 + z^2)^{3/2}} \right) \]
\[
\frac{1}{4\pi} \left( \frac{(x^2 + y_a^2)(z^2 + y^2)}{(x^2 + y_a^2 + z^2)^{3/2}} \right) \]

\[ B_y (x, y, z) = \]
\[
\frac{1}{4\pi} \left( \frac{(x_a^2 + y^2)(y_a^2 + z^2)}{(x_a^2 + y^2 + z^2)^{3/2}} \right) \]
\[
\frac{1}{4\pi} \left( \frac{(x^2 + y_a^2)(z^2 + y^2)}{(x^2 + y_a^2 + z^2)^{3/2}} \right) \]

\[ B_z (x, y, z) = \]
\[
\frac{1}{4\pi} \left( \frac{(x_a^2 + y^2)(y_a^2 + z^2)}{(x_a^2 + y^2 + z^2)^{3/2}} \right) \]
\[
\frac{1}{4\pi} \left( \frac{(x^2 + y_a^2)(z^2 + y^2)}{(x^2 + y_a^2 + z^2)^{3/2}} \right) \]
\[ B_z(x, y, z) = \]
\[ \frac{1}{4\pi} \left( \frac{R + y}{\sqrt{(R-x)^2 + (R-y)^2 + \left( \frac{1}{2} + z \right)^2}} - \frac{R - y}{\sqrt{(R-x)^2 + (R-y)^2 + \left( \frac{1}{2} + z \right)^2}} \right) \]
\[ + \frac{R - x}{\sqrt{(R-x)^2 + (R-y)^2 + \left( \frac{-1}{2} + z \right)^2}} + \frac{R + x}{\sqrt{(R-x)^2 + (R-y)^2 + \left( \frac{-1}{2} + z \right)^2}} \]
\[ \frac{R - x}{\sqrt{(R-x)^2 + (R-y)^2 + \left( \frac{-1}{2} + z \right)^2}} + \frac{R + x}{\sqrt{(R-x)^2 + (R-y)^2 + \left( \frac{-1}{2} + z \right)^2}} \]
\[ \frac{R + y}{\sqrt{(R-x)^2 + (R-y)^2 + \left( \frac{1}{2} + z \right)^2}} + \frac{R - y}{\sqrt{(R-x)^2 + (R-y)^2 + \left( \frac{1}{2} + z \right)^2}} \]
\[ \frac{R + y}{\sqrt{(R-x)^2 + (R-y)^2 + \left( \frac{1}{2} + z \right)^2}} + \frac{R - y}{\sqrt{(R-x)^2 + (R-y)^2 + \left( \frac{1}{2} + z \right)^2}} \]
\[ \frac{R - x}{\sqrt{(R-x)^2 + (R-y)^2 + \left( \frac{-1}{2} + z \right)^2}} + \frac{R + x}{\sqrt{(R-x)^2 + (R-y)^2 + \left( \frac{-1}{2} + z \right)^2}} \]
\[ \frac{R - x}{\sqrt{(R-x)^2 + (R-y)^2 + \left( \frac{-1}{2} + z \right)^2}} + \frac{R + x}{\sqrt{(R-x)^2 + (R-y)^2 + \left( \frac{-1}{2} + z \right)^2}} \]
\[ \frac{R + y}{\sqrt{(R-x)^2 + (R-y)^2 + \left( \frac{1}{2} + z \right)^2}} + \frac{R - y}{\sqrt{(R-x)^2 + (R-y)^2 + \left( \frac{1}{2} + z \right)^2}} \]
Reference
[2] Honeywell Solution Metglas 2705M
   http://www.metglas.com/products/page5_1_2_5.htm