<table>
<thead>
<tr>
<th>Lecture</th>
<th>Date and time</th>
<th>Place</th>
<th>Topic</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Oct. 2, 4:30 - 6:30 pm</td>
<td>CUHK L4 Science Center</td>
<td>Overview: quant. dynamics &amp; classical expression</td>
</tr>
<tr>
<td>2</td>
<td>Oct. 6 4 - 6 pm</td>
<td>HKUST LT H, Chen Kuan Cheng Forum</td>
<td>Spintronics: from spin scattering to circuit</td>
</tr>
<tr>
<td>3</td>
<td>Oct. 16 4:30 - 6:30 pm</td>
<td>CUHK L4 Science Center</td>
<td>Quantum metrology: entanglement &amp; measurement</td>
</tr>
<tr>
<td>4</td>
<td>Oct. 21 4 - 6 pm</td>
<td>HKU LT B, Chow Yei Ching Bldg</td>
<td>Quantum operations and control</td>
</tr>
</tbody>
</table>
Outline of Lecture 3

- Phase estimation
- Beam splitter and quantum transformation
- Metrology
  - Interferometry
  - Precision limits
- Imaging
Relation between QM Foundation and Quantum Technology

Review
Both foundational issues and technology involve connection between quantum properties and classical expression

QM Concepts
1. Entanglement
2. Quantum measurement
3. Decoherence
4. Control

Foundational Problems
1. EPR paradox and Bell's theorems
2. Schrödinger's cat
3. Interpretations of QM - informatics approach

Quantum Technology
1. q information (telecom)
2. q metrology, imaging, games, ... new ideas?
3. q computation
4. Simulation & Emulation
**Review**

Bipartite Entanglement

Quantum system 1 states: $|+\rangle, |-\rangle$

Quantum system 2 states: $|J\rangle$

Entanglement superposition: $\beta|+\rangle|J^+\rangle + \alpha|\rangle|J^-\rangle$

Consequences

Decoherence Coherence of system 1

$$\langle J^+ | J^- \rangle \rightarrow 0$$

von Neumann projective measurement

QS1 states - eigenstates of a physical observable

QS2 states in entanglement magnify the separation of the QS1 states

Resulting in projection into one of the eigenstates

e.g. Stern-Gerlach
Quantum Technology

1. Quantum information
   a. Physical basis (“Information is physical”)
   b. q communication already in existence

2. Intermediate technology - a few qubits
   a. Metrology - high precision
   b. Sensors - high sensitivity
   c. Imaging - high resolution
   d. Quantum games and toys - not specific
   e. Switches
   f. Needs revolutionary ideas!

3. q computation (much written about)

4. Emulator
   a. Atomic lattices to simulate difficult problems in solids (strongly correlated effects).
   b. Excitons in coupled quantum wells.
1. Find the parameter ($\varphi$) of a state with a known form.

2. A sequence of data (A) of the measured probability distribution of the state over the eigenstates of an observable.

3. Deduce the parameter in a probability distribution.

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Bayes’ Theorem

$$P\{\varphi|A\} = P\{A|\varphi\}P\{\varphi\}/P\{A\}$$

Conditional probability

$$P\{A\} = \int_{0}^{\pi} P\{A|\varphi\}P\{\varphi\}$$

Bayesian hypothesis

$$P\{\varphi\} = 1/\pi, \text{ for } 0 \leq \varphi \leq \pi$$

---

Examples

- Coherent state $|\varphi\rangle = e^{i\varphi} |+\rangle + |-\rangle$

$$A = [u, u, d, u, \ldots]$$

$$P\{A|\varphi\}$$

$$P\{\varphi|A\}$$

---

C. Helstrom, Quantum Detection and Estimation Theory (Academic Press, NY, 1976); also Peres
Mach-Zehnder interferometer

Parameter of state $\varphi$

Single photon input $|a\rangle$

Interference intensity $M$

$P\{c|\varphi\} = \sin^2 \frac{\varphi}{2}$

$M = P\{d|\varphi\} - P\{c|\varphi\} = \cos \varphi$

$e^{i\varphi}|a\rangle + i|b\rangle$

$P\{d|\varphi\} = \cos^2 \frac{\varphi}{2}$

J. P. Dowling, Contemp. Physics, 49, 125 (2008)
Unitary transformations

$$\begin{bmatrix} a_+^\dagger, b_+^\dagger \end{bmatrix} = \begin{bmatrix} a_i^\dagger, b_i^\dagger \end{bmatrix} \begin{bmatrix} t_a & r_a \\ r_b & t_b \end{bmatrix} \quad M = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Mirror

Homework: 1D scattering s-matrix

Beam Splitter S and Mirror M

Interference from transformations

$$SU(\varphi)MS \begin{bmatrix} 1 \\ 0 \end{bmatrix} = i e^{i\varphi/2} \begin{bmatrix} \cos \frac{\varphi}{2} \\ \sin \frac{\varphi}{2} \end{bmatrix}$$

Gerry and Knight, Introductory Quantum Optics (CUP)
Hong-Ou-Mandel Effect

Photon input state: $a_i^\dagger b_i^\dagger |0\rangle = |1,1\rangle$

Beam splitter transformation

$$ (\hat{S}a^\dagger)(\hat{S}b^\dagger) = \frac{1}{2} (ia^{\dagger 2} + b^\dagger a^\dagger - a^\dagger b^\dagger + ib^{\dagger 2}) $$

$$ = \frac{i}{2} (a^{\dagger 2} + b^{\dagger 2}) $$

Cancellation because photons are bosons, cf. neutrons (fermions).

Output state: $(a_i^{\dagger 2} + b_i^{\dagger 2}) |0\rangle = |2,0\rangle + |0,2\rangle$

Entangled state

Gerry and Knight, Introductory Quantum Optics (CUP)
Quantum Metrology

An example of “intermediate” quantum technology

High precision phase measurement

\[ \Delta \varphi = \frac{1}{\Delta n} = \frac{1}{\sqrt{n}} \]

Shot noise limit

\[ \Delta \varphi = \frac{1}{n} \]

Heisenberg limit

Squeezed vacuum in B (Caves)

In practice, closer to SNL

N00N state

Vacuum state in B -> noise (Caves)

When \( \varphi = 0 \), C is dark, D is light.

J. P. Dowling, Contemp. Physics, 49, 125 (2008) “Q optical metrology - the lowdown on high-N00N states”.

LJSham 08/09/25-08/10/24
Shot Noise Limit

Uncertainty in intensity measurement

\[ \Delta M \propto \Delta n \]

Laser ~ coherent state

\[ D(\alpha) = \exp(\alpha a^\dagger - \alpha^* a) = e^{-|\alpha|^2/2} e^{\alpha a^\dagger} e^{-\alpha^* a} \]

\[ |\alpha\rangle \equiv D(\alpha) |0\rangle = e^{-|\alpha|^2/2} e^{\alpha a^\dagger} |0\rangle \]

Uncertainty relation (Dirac)

\[ \Delta n \Delta \varphi \sim 1 \]

Shot noise limit

\[ \Delta \varphi = \frac{1}{\Delta n} = \frac{1}{\sqrt{n}} \]

define observable for phase??

Rough “derivation”

\[ \Delta E \Delta t \sim \hbar \]

More precise

\[ \hat{x} = \frac{1}{\sqrt{2}} (a + a^\dagger); \quad \hat{p} = \frac{1}{i\sqrt{2}} (a - a^\dagger) \]

J. P. Dowling, Contemp. Physics, 49, 125 (2008)
Heisenberg Limit

Squeezed state

\[ S(\xi) = \exp\left[\frac{1}{2} \xi^* a^2 - \frac{1}{2} \xi (a^\dagger)^2\right], \quad \xi = re^{i\theta} \]

\[ |\alpha, \xi\rangle = D(\alpha)S(\xi)|0\rangle \]

Squeezed vacuum

\[ \Delta\eta \sim e^{-r}; \quad \Delta\xi \sim e^r \]

Squeezed \(|\alpha, r\rangle\)

Heisenberg limit

\[ \Delta\varphi = \frac{1}{n} \]

Vacuum state in B -> noise (Caves)

\[ |1, 0\rangle \rightarrow t_a |1,0\rangle + r_a |0,1\rangle \]

Use squeezed vacuum - experimentally slightly below SNL

The N00N state

Mach-Zehnder Interferometer

\[ |N, 0\rangle + |0, N\rangle \rightarrow e^{iN\phi} |N, 0\rangle + |0, N\rangle \]

Interference intensity \( M_{N00N}(\phi) = I_A \cos(N\phi) \)

Phase shifter on A

QC circuit

Sensitivity

\[ \Delta \phi = \frac{1}{N} \]

J. P. Dowling, Contemp. Physics, 49, 125 (2008) “Q optical metrology - the lowdown on high-N00N states”.
Generation of the N00N state

Hong-Ou-Mendel expt

Via the Kerr medium

Projective measurement

intermediate application of QC

J. P. Dowling, Contemp. Physics, 49, 125 (2008)
Photon phase gate by CQED

- **Qubit by polarization**
  \[ |X>, |Y> \]

- **Cavity Modes**

- **Gate Transformation**
  \[
  \begin{align*}
  |XX> & \rightarrow e^{i\phi} |XX> \\
  |XY> & \rightarrow |XY> \\
  |YX> & \rightarrow |YX> \\
  |YY> & \rightarrow |YY>
  \end{align*}
  \]

- **Linear reflection?**
  Reduced by EIT which also yields laser cooling.

- **Nonlinearity?**
  Coupling to dot, cf Kerr medium

Yao, Liu and Sham, PRL 92, 217402 (2004).
Future challenges of the N00N state

N00N state is fragile against photon fluctuation

\[ a^\dagger (|N, 0\rangle + |0, N\rangle) = |N, 0\rangle \]

M&N state

\[ \langle M|_A \langle N|_B + \langle N|_A \langle M|_B \]

Interference pattern complicated

Measurement

J. P. Dowling, Contemp. Physics, 49, 125 (2008) “Q optical metrology - the lowdown on high-N00N states”.

LJSham 08/09/25-08/10/24
Quantum Imaging

Application of spatially resolved modes

Imaging with entangled photons

Down conversion to two photons

Holes stenciled in disc

XY recorder of position of fiber tip

Pittman et al. PRA 52, R3429 (1995)

Lloyd et al. Science 306, 1330 (2004);
Quantum Imaging

Application of spatially resolved modes

High resolution

Ryleight limit

\[ \Delta x = \frac{\lambda}{2\pi} \Delta \varphi = \frac{\lambda/2\pi}{\sqrt{n}} \]

Heisenberg limit

\[ \Delta x = \frac{\lambda/2\pi}{n} \]

N00N state also applicable

Dowling et al.

Pittman et al. PRA 52, R3429 (1995)

Lloyd et al. Science 306, 1330 (2004);
Quantum measurements of a single spin

• Basic Problem
  – Optical probe of a spin produces at most one photon at one shot.

• Current practice
  – Periodic pump & probe and collection of all outgoing photons for change in transmission

• Application of quantum optics methods
  – Cavity QED (Renbao Liu, Wang Yao, LJS)
  – Cycling transitions (used in ion traps)
    • Utilizing levels in the same dot (Steel)
    • Constructing coupled dots for suitable levels (Gammon)
  – Noise spectra (Steel): $T_2^*$

• Periodic weak measurement and data processing (correlation) of the stochastic time series of data (continue in the next slide)
Weak measurements of Faraday rotation by a single spin

- A spin initially unpolarized
- Periodic optical pulses at interval $\tau$
- Faraday rotation of the coherent light polarization by the spin
  - Light $|+\theta\rangle$ for spin up and $|−\theta\rangle$ for spin down
- Polarized beam splitter (PBS)
  - $|+\theta\rangle$ pass; $|−\theta\rangle$ prob(reflect) =$D$, prob(pass) = 1-$D$.
  - POVM measurement operators:
    $E(\text{reflect}) = D | \downarrow \rangle \langle \downarrow |$  
    $E(\text{pass}) = | \uparrow \rangle \langle \uparrow | + (1 - D) | \downarrow \rangle \langle \downarrow |$
- Photon counts of successful measurement of spin in $|\downarrow\rangle$ state occur stochastically (like shot noise)
- $\theta \sim 10$ micro-radian
- Pulse amplifies $\theta$ to $D \sim N \theta^2$

How to extract pure decoherence time from data

Data analysis of the correlations in the time series of random events occurring between dephasing precession (FID)

Monte Carlo simulation (d) $10^{10}$ pulses
(a) $1/T_2 = 0$, inhomogeneity $\sigma = 0$
(b) $T_2 = 200\text{ns}$, $\sigma = 0$; $g^{(2)}(t) \Rightarrow T_2$
(c) $T_2 = 200\text{ns}$, $1/\sigma = 10\text{ns}$; $g^{(2)}(t) \Rightarrow T_2^*$

Third order correlation $t_1 = t_2$ contour $\Rightarrow T_2$

$D = 0.3 \times 10^{-4}$

$G^{(3)}(t_1, t_2) \equiv \left\langle g^{(3)}(t_1, t_2) \right\rangle - \left\langle g^{(2)}(t_1) \right\rangle \left\langle g^{(2)}(t_2) \right\rangle$

contains

$$\frac{1}{2} e^{-(t_1 + t_2)/T_2} e^{-\sigma^2(t_1 - t_2)^2/2} \cos \left(\omega_0(t_1 - t_2)\right)$$

period: pulse $\tau = 0.3\text{ ns}$; Larmor = $3\text{ ns}$
Summary

• Entanglement can increase the precision of phase estimation
• Entanglement can enhance imaging
• Methods developed for quantum computing can be used for “intermediate” quantum technology
• We hope for the reverse as well.
• Concepts perhaps not taught in the usual quantum mechanics course
  • Quantum formulation of interferometry
  • Use of entanglement and other quantum light states
  • Generation of entangled states
  • Quantum limit of measurement precision beyond the uncertainty principle
• Error analysis