Is the Universe Rotating?

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Toutatis’ spin axis traces a curve around the asteroid’s surface once every 5.41 days. 
Animation courtesy NASA

Jupiter rotates once every ~10hrs. 
Animation courtesy NASA

Sun rotates once every ~25 days. 
Animation by mc²

Milky Way is rotating, with period ~220 MYr at Sun’s orbit.
Even clusters of galaxies rotate → discovery of dark matter

Photo credit: Digitized Sky Survey, Palomar Observatory, STScI
Is the Universe Rotating?

I. Some history: Plato, Copernicus, Newton, Gamow, Gödel, Li, Barrow, Hawking

II. Recent controversy: AOE – is there a preferred direction in the universe?

III. Our own work: constraining the rotational speed of the universe
Summary

I. If the universe rotates:
- May explain the rotation of most stellar objects
- But may violate causality
- May impact our understanding of time (time travel possible?)
- May need to throw away inflation theory, revise standard cosmological theory

II. Using data to constrain rotation speed: highly dependent on model, not conclusive yet
Plato-Aristotle Cosmology

- Plato: heavenly objects only take up perfect motion = circular motion with uniform speed
- Aristotle: Earth sits still at the center of the universe → a finite, rotating universe; human is unique
張衡在《靈憲》（渾天說）：天體于陽，故圓以動，地體于陰，故平以靜。

Looking North: star trails, rotation of the celestial sphere
Copernicus’ Cosmology (simplified version)

- More economical to have the earth rotating
- Stars and the sun are fixed; Earth and other planets orbit around the sun

Revolution!

Nicolaus Copernicus
Newtonian Cosmology

- Space is absolute and static
- Unstable if the universe is finite: forces not balanced anywhere except at the exact center, but unstable equilibrium even there.

\[ m = 1 \]

Sir Isaac Newton

Newton: the universe must be infinite and static! → no rotation!
Rotation to the rescue (partially)!

Stockum (1937), Gödel (1949):

Uniform density, rigidly rotating Newtonian universe

\[ \nabla^2 \Phi = 4 \pi G \rho, \quad \to \quad \Phi = \pi G \rho (x^2 + y^2) = \pi G \rho r^2, \]

\[ \therefore \text{Gravitational force towards rotation axis:} \quad \vec{f}_{\text{grav}} = -\nabla \Phi = -2 \pi G \rho \hat{r}, \]

Centrifugal force: \[ \vec{f}_{\text{cent}} = \Omega^2 \hat{r}, \]

\[ \Omega^2 = 2\pi G \rho \text{ (both constant) } \to \text{universe static everywhere!} \]

Coriolis force = \[ \vec{f}_{\text{Cor}} = 2\hat{\nu} \times \hat{\Omega} \text{ same everywhere} \]

Homogeneity, but anisotropic


Cosmic Microwave Background
Origin of elements
Alpha decay

- rotation of stellar objects caused by rotation of the universe
- Rotation of the universe can be observed
- A solution of GR corresponding to rotating universe can probably be constructed.

http://www.aip.org/history/ohilist/4325.html
**Kurt Gödel**

**Gödel’s Incomplete Theorems (1930)**

http://en.wikipedia.org/wiki/G%C3%B6del%27s_incompleteness_theorems

‘In any formal system adequate for number theory there exists an undecidable formula - that is, a formula that is not provable and whose negation is not provable.’

‘the consistency of a formal system adequate for number theory cannot be proved within the system.’


**Fundamental limitations of formal logic systems (cf. Uncertainty Principles, Relativity)**

Einstein ‘came to the Institute merely in order to have the privilege of walking home with Gödel.’ – O. Morgenstern

Gödel’s Rotating, Static Universe (1949)

\[ ds^2 = a^2[-d\tau^2 + dx^2 - (e^{2x}/2)dy^2 + dz^2 - 2e^x d\tau dy] \]

There exists closed time-loops: after completing an orbit, one goes back to the starting point at an earlier time!

Light cones in standard cosmology

Gödel’s universe at rotation axis

A universal time with future all pointing in the direction of decreasing density/temperature

Futures are not ‘parallel’

At large \( r \), fast particles go backward in time

http://en.wikipedia.org/wiki/G%C3%B6del_metric
Gödel’s Rotating, Static Universe (1949)

- Gödel’s birthday gift to Einstein
- Exact solution of GR, motivated by Gamow’s paper
- Violates causality! One can go back to kill one’s father/mother!
- Einstein’s response: Nature must have ways to prevent Gödel’s space-time from materializing
- Static → considered to be just a toy model, largely ignored
- Modern view of physics: if it’s not forbidden by some principles, it should be there! Could there be many Gödel universes? Ours is just an accident?
- Time is just an illusion?

But Gamow’s conjecture is still lurking around ...

http://www.aip.org/history/ohilist/4325.html
Empirical relation between angular momentum ($J$) and mass of galaxies ($M$):

\[ J = k M^{5/3} \]

$k \sim 0.4$ (cgs units)

valid also for many other celestial objects!

Carrasco et al. (1982).

### Table

<table>
<thead>
<tr>
<th>Class of objects</th>
<th>$j \propto M^\alpha$</th>
<th>$Q \propto M^\beta$ Corrected for density effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asteroids, satellites, and planets</td>
<td>0.66 ± 0.06</td>
<td>0.66 ± 0.04</td>
</tr>
<tr>
<td>Contact binaries</td>
<td>0.71 ± 0.05</td>
<td>0.76 ± 0.02</td>
</tr>
<tr>
<td>Visual binaries</td>
<td>0.67 ± 0.07</td>
<td>0.68 ± 0.03</td>
</tr>
<tr>
<td>Clusters, bulges, and elliptical galaxies</td>
<td>0.66 ± 0.07</td>
<td>0.62 ± 0.01</td>
</tr>
<tr>
<td>Spiral galaxies*</td>
<td>0.72 ± 0.05</td>
<td>—</td>
</tr>
<tr>
<td>Spiral galaxies and super cluster</td>
<td>0.84 ± 0.04</td>
<td>0.74 ± 0.03</td>
</tr>
<tr>
<td>All objects</td>
<td>0.94 ± 0.09</td>
<td>0.71 ± 0.01</td>
</tr>
</tbody>
</table>

\* Vettolani et al. (1980)

Effect of the Global Rotation of the Universe on the Formation of Galaxies
Li-Xin Li¹,²

Received July 1, 1997. Rev. version October 1, 1997

The effect of the global rotation of the universe on the formation of galaxies is investigated. It is found that the global rotation provides a natural origin for the rotation of galaxies, and the morphology of the objects formed from gravitational instability in a rotating and expanding universe depends on the amplitude of the density fluctuation, different values of the amplitude of the fluctuation lead to the formation of elliptical galaxies, spiral galaxies, and walls. The global rotation gives a natural explanation of the empirical relation between the angular momentum and mass of galaxies: \( J \propto M^{5/3} \). The present angular velocity of the universe is estimated at \( \Omega \sim 10^{-13} \text{ rad yr}^{-1} \).

Physics: Coriolis force + gravitational collapse + expanding, rotating background

Similar to formation of typhoons!

- Derived \( J \propto M^{5/3} \)
based on formation of galaxies in a rotating universe, and conservation of angular momentum.

- Explained two types of galaxies (spiral, elliptical).

- Estimated the rotation speed of the universe by comparing with data: \( \Omega \sim 10^{-13} \text{ rad yr}^{-1} \)
Are these both due to Coriolis force?

A pseudo force due to the rotating frame

Images courtesy NASA
CMB Constraint of Rotating Universe

- Treat the rotation as perturbations; use CMB data to constrain rotation speed $\Omega$
  - For closed universe (Hawking 1969)
    - $< 10^{-14} - 7 \times 10^{-17}$ rad/yr
  - For open universe (Hawking 1969)
    - $< 2 \times 10^{-17}$ rad/yr
  - For flat universe (Barrow et al. 1985)
    - $< 1.5 \times 10^{-15}$ rad/yr

Bianchi models (1st order effect) uniform $\Omega$


Inflation model: exponential expansion of the universe dampens out any initial rotation: $\Omega = 0$!

Cosmic Microwave Background (CMB) Anisotropies

Emitted at $t \sim 400,000$ yrs, farthest EM signals we can observe!

$T = 2.725 \pm 10^{-5} \text{ K}$

CMB Anisotropies = Temperature fluctuations $\sim 10^{-5} \text{ K}$

Figure courtesy NASA/WMAP
Axis Of Evil

Is there a preferred direction in the universe?
Data analysis: 

D. J. Schwarz et al., PRL 93, 221301 (2004).
De Oliveira-Costa et al., PRD 69, 063516 (2004).

Theory:

‘Axis of Evil’?

Galactic coordinates: 
(l ~-90°, b ~ 50°)

longitude latitude

Fig. 2. Right ascensions of the alignments discussed in the text. The WMAP dipole, quadrupole, and two of the octopoles are indicated. NGP is the North Galactic pole. EQX are the equinoxes. The axis of the spiral galaxy spin asymmetry from [8] is also shown. The declinations of all these alignments are within about ±15° of each other and are ~0°.
Milky Way
AOE points towards Virgo Cluster

distance ~ $6 \times 10^7$ l.y.s, > 2000 galaxies. Milky Way is being drawn there at several hundred km/s.

Photo credit: Digitized Sky Survey, Palomar Observatory, STScI
Table I. Number counts and net asymmetries for the R4 ranges indicated. The 3rd row gives the combined numbers for the first two rows. The last column gives the number of standard deviations for the asymmetries.

<table>
<thead>
<tr>
<th>RA Range</th>
<th>( N^+ )</th>
<th>( N^- )</th>
<th>( N_{\text{Tot}} )</th>
<th>((N^+ - N^-)/N_{\text{Tot}} \pm \sigma)</th>
<th>( \langle A \rangle / \sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(R) 80° to -80°</td>
<td>118</td>
<td>104</td>
<td>222</td>
<td>0.063±0.067</td>
<td>+0.94</td>
</tr>
<tr>
<td>(L) 150° to 210°</td>
<td>296</td>
<td>368</td>
<td>664</td>
<td>-0.108±0.039</td>
<td>-2.79</td>
</tr>
<tr>
<td>(R-L) Combined</td>
<td>178</td>
<td>264</td>
<td>886</td>
<td>-0.0971±0.0336</td>
<td>-2.89</td>
</tr>
</tbody>
</table>

Prob. \(< 0.39\%\)

Parity violation?
Project Galaxyzoo
http://www.galaxyzoo.org/Project2.aspx

Count the spirals; determine whether there’s AOE!
http://news.bbc.co.uk/2/hi/science/nature/6289474.stm
Finally, our work ...

Our Model

\[ ds^2 = a^2(\eta) \left\{ [1 - f(r, \eta)] d\eta^2 + [1 - h(r, \eta)] dr^2 - [1 - h(r, \eta)] r^2 d\theta^2 \right\} - [1 - k(r, \eta)] dz^2 + 2r^2 a(\eta) \Omega(r, \eta) d\theta d\eta \],

Add rotational perturbations to the flat RW model (standard model)

- \( \Omega(r, \eta) \): angular velocity
  - Allows non-uniform rotation
- Flat RW model
  - Supported by observation
- 2\textsuperscript{nd} order perturbations
  - Parity symmetry
  \( f(r, \eta), h(r, \eta) \) and \( k(r, \eta) \)

**Constraint:** \( \Omega < 10^{-9} \text{ rad yr}^{-1} \) at CMB emission
**CMB Constraint of Rotating Universe**

- Treat the rotation as perturbations; use CMB data to constrain rotation speed $\Omega$
  - For closed universe (Hawking 1969)
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    - $< 1.5 \times 10^{-15}$ rad/yr

**Bianchi models**

- $1^{st}$ order effect)
- uniform $\Omega$

**RW model**

- $2^{nd}$ order) $\Omega(r)$

- For flat $\Lambda$CDM universe (Su and Chu 2009)
  - $< 10^{-9}$ rad/yr

Cf. Li: $\Omega \sim 10^{-13}$ rad/yr

Looser bound: could we revive Gamow, Li’s dream?

Explain AOE?

...to be continued...
Summary

I. If the universe rotates:
- May explain the rotation of most stellar objects
- But may violate causality
- May impact our understanding of time (time travel possible?)
- May need to throw away inflation theory, revise standard cosmological theory

II. Using data to constrain rotation speed: highly dependent on model, not conclusive yet
Is the Universe Rotating?

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Newtonian Cosmology

- A spherical universe of uniform density
  \( \rho = \frac{3M}{4\pi R^3} \)
- Dynamics of a unit mass on ‘surface’:

\[
\frac{1}{2} \left( \frac{\dot{R}}{R} \right)^2 = \frac{4\pi G}{3} \rho + \frac{\eta}{R^2}
\]

\[
\frac{\ddot{R}}{R} = -\frac{4\pi G}{3} \rho
\]

- \( \eta < 0 \): expansion will slow down to stop, and then the universe collapses
- \( \eta = 0 \): expands exactly at the "escape velocity"; expansion will slow down but continues forever
- \( \eta > 0 \): expansion continues forever
Robertson-Walker Metric

- Most general metric (non-rotating)
  - Homogeneity and isotropy
    \[ g_{\mu\nu} dx^\mu dx^\nu = -dt^2 + a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right] \]
  - \( k = -1 \) (open), \( 0 \) (flat), \( 1 \) (closed)
  - \( a(t) \) : scale factor

[Diagram showing spatial geometries]
Mach’s Principle

BUT: rotation with respect to what?

Rotation w.r.t. space is ok if space is absolute (Newtonian)!

Mach’s Principle: motion is meaningful only relative to other bodies; ~ ‘the inertia of a body is determined in relation to all other bodies in the universe’

There’s nothing outside of the universe, no other reference!

Uniform rotation of the universe violates Mach’s Principle!

Water surface curved when the bucket is rotating.
Newton: rotation is relative to absolute space.
Mach: rotation is relative to distant stars.
Rotation in General Relativity

In Newtonian Physics space is absolute, ok to talk about rotation of matter w.r.t. space

GR: space is relative, deformable by matter/energy

Invariant interval:
\[ ds^2 = dt^2 - dx^2 - dy^2 - dz^2 \]

Flat non-rotating space-time
\[ ds^2 = dt^2 - dr^2 - r^2 d\theta^2 - dz^2 \]

Polar coordinate
\[ ds^2 = dt^2 - dr^2 - r^2 d\theta^2 - dz^2 \]

Rotating with angular speed \( \Omega \):
\[ ds^2 = dt^2 - dr^2 - r^2 (d\theta - \Omega dt)^2 - dz^2 \]
\[ ds^2 = (1 - r^2 \Omega^2) dt^2 - dr^2 - r^2 d\theta^2 - dz^2 + 2r^2 \Omega d\theta dt \]

Distortion of space-time due to rotation
Analytic Solutions of the EFEs

\[
2u_0(r, \eta) = \frac{a(\eta)}{2} \left\{ -2u_2(r, \eta) + \frac{[u_2(r, \eta)]^2}{r^2a^2(\eta)} + k(r, \eta) + T(r, \eta) \right\}
\]

\[
2u_1(r, \eta) = \frac{-a^2(\eta)}{8a^2(\eta) - 4a(\eta)a'(\eta)} \left\{ -\dot{a}(\eta)[2k'(r, \eta) + 2T'(r, \eta)] \\
+ a(\eta)[-2k'(r, \eta) + \dot{L}'(r, \eta)] \right\},
\]

\[
2\rho(r, \eta) = -\frac{1}{32\pi a^2(\eta)} \left\{ -4Aa^2(\eta)[k(r, \eta) + T(r, \eta)] \\
+ \frac{4\dot{a}(\eta)[3\dot{k}(r, \eta) - 2\dot{L}(r, \eta)]}{a(\eta)} - \frac{2[2k'(r, \eta) - L'(r, \eta)]}{r} \\
+ 12ra^2(\eta)\Omega(r, \eta)\Omega'(r, \eta) + r^2a^2(\eta)\Omega'^2(r, \eta) - 4k''(r, \eta) \\
+ 2L''(r, \eta) + 4r^2a^2(\eta)\Omega(r, \eta)\Omega''(r, \eta) \\
+ 4 \left[ A^2(\eta) + \frac{3\dot{a}^2(\eta)}{a^2(\eta)} \right] \{k(r, \eta) + T(r, \eta) \}
\]

\[
\rho = \rho' + 1\rho'' + 2\rho', \\
P = \rho' + \rho'' + 2P, \\
u^\mu = \rho' + \rho'' + 2\mu + u^\mu, \\
T(r, \eta), L(r, \eta) and k(r, \eta)
\]
Analytic Solutions of the EFEs

- \( 2P(r, \eta) \) : 3 equations for each spatial direction
  - Same: \( \Omega(r, \eta) = a^{-3}(\eta)A(r) + B(\eta) r, \eta) = \ldots \) known \( k, T, L \)
  - \( W \)
    \[
    T'(r, \eta) - rT''(r, \eta) = -\frac{2r^{3}A'^{2}(r)}{a^{4}(\eta)} - \frac{\Omega_{\Lambda} r[3A'(r) + rA''(r)]^{2}}{2\Lambda(1 - \Omega_{\Lambda})a^{3}(\eta)},
    \]
    \[
    \frac{r}{a(\eta)} \frac{d}{d\eta} [a^{2}(\eta)\dot{L}(r, \eta)] - a(\eta)[L'(r, \eta) + rL''(r, \eta)]
    \]
    \[
    = ra(\eta)T''(r, \eta) - r^{3}a^{-3}(\eta)A'^{2}(r).
    \]

- \( k \) is absent, freedom! Set \( k = 0 \)
- homogeneous rotation \( B \) is absent.
Analytic Solutions of the EFEs

- Taylor series + separation of variables

Let
\[ A(r) = \Omega_M \sum_{n=2}^{\infty} c_n r^n \quad \text{and} \quad L(r, \eta) = \Omega_M^2 \sum_{n=0}^{\infty} E_n(\eta) r^n \]

\[
T(r, \eta) = -\frac{\alpha^2 \Omega_M^2}{3a^4(\eta)} r^6 - \frac{4\Omega_\Lambda \alpha^2 \Omega_M^2}{\Lambda(1 - \Omega_\Lambda) a^3(\eta)} r^4,
\]

\[
0 = E_{2n+1}(\eta),
\]

\[
0 = \frac{1}{a(\eta)} \frac{d}{d\eta} \left[ a^2(\eta) \dot{E}_0(\eta) \right] - 4a(\eta) E_2(\eta),
\]

\[
-\frac{48\Omega_\Lambda \alpha^2}{\Lambda(1 - \Omega_\Lambda) a^2(\eta)} = \frac{1}{a(\eta)} \frac{d}{d\eta} \left[ a^2(\eta) \dot{E}_2(\eta) \right] - 16a(\eta) E_4(\eta),
\]

\[
-\frac{14\alpha^2}{a^3(\eta)} = \frac{1}{a(\eta)} \frac{d}{d\eta} \left[ a^2(\eta) \dot{E}_4(\eta) \right] - 36a(\eta) E_6(\eta),
\]

\[ \vdots \]

\[
0 = \frac{1}{a(\eta)} \frac{d}{d\eta} \left[ a^2(\eta) \dot{E}_{2n-2}(\eta) \right] - 4n^2 a(\eta) E_{2n}(\eta).
\]

\[ \rho = \rho^\rho + \rho^\mathcal{P} + 2\rho^\mathcal{A}, \]

\[ P = \rho^\mathcal{P} + \rho^\mathcal{A} + 2\rho^\mathcal{P}, \]

\[ w^\mu = \rho^\mathcal{P} + \rho^\mathcal{A} + 2\rho^\mathcal{P}, \]

\[ T(r, \eta), L(r, \eta) \text{ and } k(r, \eta) \]

Recursion relation!
Its 2nd-Order Sachs-Wolfe Effects

- **1st order:**
  \[
  \frac{\delta T_O}{T_O} = -\frac{1}{2} g_{00} |^\xi_0 + o k^2 (g_{02} + u_2) |^\xi_0 - 1 k^0 |^\xi_0
  \]
  \[
  = 0 \quad \text{Expected!}
  \]

- **2nd order:**
  \[
  \frac{\delta T_O}{T_O} = \left[ -\frac{2u_1(r_\lambda, \eta_\lambda)}{a(\eta_\lambda)} \sin \phi + \frac{2u_0(r_\lambda, \eta_\lambda)}{a(\eta_\lambda)} + 2 k^0(\lambda) + \frac{1u_2(r_\lambda, \eta_\lambda)}{a(\eta_\lambda)} k^2(\lambda) \right] |_{\eta_0}^{\eta_\epsilon}
  \]
  \[
  = \int_{\eta_\epsilon}^{0} \left[ -\frac{\dot{T}(-\lambda \sin \phi, \lambda)}{2} + T'(-\lambda \sin \phi, \lambda) \sin \phi - \frac{\ddot{L}(-\lambda \sin \phi, \lambda)}{2} \sin^2 \phi \right] d\lambda
  \]
  \[
  - \frac{1}{2} \left\{ \frac{\Omega_\Lambda^2 a^4(\eta_\epsilon)}{4\Lambda^2 (1 - \Omega_\Lambda)^2} [3\Omega'(r_\epsilon, \eta_\epsilon) + r_\epsilon \Omega''(r_\epsilon, \eta_\epsilon)]^2 + T(r_\epsilon, \eta_\epsilon) \right\}
  \]
  \[
  + \frac{\Omega_\Lambda \sin \phi}{2\Lambda (1 - \Omega_\Lambda)} \left[ 2\dot{a}(\eta_\epsilon) T'(r_\epsilon, \eta_\epsilon) - a(\eta_\epsilon) \dot{L}'(r_\epsilon, \eta_\epsilon) \right],
  \]

**What we need!!**
Constraints on Our Model

- Simplest case (homogeneous rotation):
  \[ \Omega(r, \eta) = B(\eta) \]
  - 2nd order SW effect:
    \[ \frac{\delta T_\Omega}{T_\Omega} = 0 \]
    \[ u^2(r, \eta) = \Omega(r, \eta) \]
  - Only the difference is important!

Effect of the rotating metric \(\leftrightarrow\) Cancel

Relativistic Doppler effect caused by the sources rotating in a stationary metric
Constraints on Our Model

• **Second Trial:** \( \Omega(r, \eta) = \Omega_M a^3(\eta_c) \frac{r^2}{(r^2 a^3(\eta))} \)

\[
\frac{\delta T_O}{T_O} = a_2 \sin^2 \phi + a_4 \sin^4 \phi + a_6 \sin^6 \phi
\]

\[
= A_0 Y_0^0(\phi, \theta) + A_2 Y_2^0(\phi, \theta) + A_4 Y_4^0(\phi, \theta) + A_6 Y_6^0(\phi, \theta).
\]

• **Cannot explain alignments of multipoles in our example**
• **With** \( \Omega_\Lambda = 0.742, H_0 = 71.9 \text{km/s/Mpc} \) and \( A_n \sim 10^{-5} \)
• **Constraint** \( \sim 10^{-9} \text{ rad yr}^{-1} \)

  – At the last scattering surface
  – When photons decoupled

Examination of Evidence for a Preferred Axis in the Cosmic Radiation Anisotropy

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(Received 22 February 2005; published 11 August 2005)

We examine previous claims for a preferred axis at \((b, l) \approx (60, -100)\) in the cosmic radiation anisotropy, by generalizing the concept of multipole planarity to any shape preference. Contrary to earlier claims, we find that the amount of power concentrated in planar modes for \(l = 2, 3\) is not inconsistent with isotropy and Gaussianity. The multipoles’ alignment, however, is indeed anomalous, and extends up to \(l = 5\) rejecting statistical isotropy with a probability in excess of 99.9%. There is also an uncanny correlation of azimuthal phases between \(l = 3\) and \(l = 5\). We are unable to blame these effects on foreground contamination or large-scale systematic errors. This reappraisal may be crucial in identifying the theoretical model behind the anomaly.

FIG. 1 (color online). The \(l = 5\) multipole in galactic coordinates (top) and aligned with \((b, l) \approx (50, -91)\) (middle), and the \(l = 3\) multipole in its preferred frame. We have superposed the multipole vectors, and the chain linking them.