Chapter 3: Dielectric Waveguides and Optical Fibers
The Nobel Prize in Physics 2009 was divided, one half to Charles Kuen Kao "for groundbreaking achievements concerning the transmission of light in fibers for optical communication", the other half jointly to Willard S. Boyle and George E. Smith "for the invention of an imaging semiconductor circuit – the CCD sensor".
Professor Charles Kao, who has been recognized as the inventor of fiber optics, left, receiving an IEE prize from Professor John Midwinter (1998 at IEE Savoy Place, London, UK; courtesy of IEE).

Professor Charles Kao served as Vice Chancellor (President) of The Chinese University of Hong Kong from 1987 to 1996.

“The introduction of optical fiber systems will revolutionize the communication network. The low-transmission loss and the large bandwidth capability of the fiber system allow signals to be transmitted for establishing communication contacts over large distances with few or no provisions of intermediate amplification.” —Charles K. Kao
Dr. Charles Kuen Kao is a pioneer in the use of fiber optics in telecommunications. He is recognized internationally as the “Father of Fiber Optic Communications”.

He was born in Shanghai on November 4, 1933, and was awarded BSc in 1957 and PhD in 1965, both in electrical engineering, from the University of London.

He joined ITT in 1957 as an engineer at Standard Telephones and Cables Ltd., an ITT subsidiary in the United Kingdom.

In 1960, he joined Standard Telecommunications Laboratories Ltd., UK, ITT’s central research facility in Europe. It was during this period that Dr. Kao made his pioneering contributions to the field of optical fibers for communications. After a four years’ leave of absence spent at The Chinese University of Hong Kong, Kao returned to ITT in 1974 when the field of optical fibers was ready for the pre-product phase.

In 1982, in recognition of his outstanding research and management abilities, ITT named him the first ITT executive scientist.

From 1987 until 1996, Dr. Kao served as vice chancellor (president) of The Chinese University of Hong Kong.
Professor Charles Kao
Engineer and Inventor of Fibre Optics
Father of Fiber Optic Communications

He received L.M. Ericsson International Prize, Marconi International Fellowship, 1996 Prince Philip Medal of the Royal Academy of Engineers*, and 1999 Charles Stark Draper Prize**. He was elected a member of the National Academy of Engineering of USA in 1990.

*The Royal Academy of Engineering Medals
The Fellowship of Engineering Prince Philip Medal (solid gold) “For his pioneering work which led to the invention of optical fibers and for his leadership in its engineering and commercial realization; and for his distinguished contribution to higher education in Hong Kong”.

**The Royal Academy of Engineering Medals
The development of optical fiber technology was a watershed event in the global telecommunications and information technology revolution. Many of us today take for granted our ability to communicate on demand, much as earlier generations quickly took for granted the availability of electricity. But this dramatic and rapid revolution would simply not be possible but for the development of silica fibers as a high bandwidth, light-carrying medium for the transport of voice, video, and data. The silica fiber is now as fundamental to communication as the silicon integrated circuit is to computing. Optical fibers are the “concrete” of the “information superhighway.” By the end of 1998, there were more than 215 million kilometers of optical fibers installed for communications worldwide. Through their efforts, Kao, Maurer, and MacChesney created the basis of modern fiber optic communications. Their creative application of materials science and engineering and chemical engineering to every aspect of fiber materials composition, characterization, and manufacturing, their understanding of the stringent materials requirements placed on the fiber by the performance needs of the telecommunications system, and, above all, their dedication to achieving their vision, were all critical to their success.
Chapter 3: Dielectric Waveguides and Optical Fibers

√ Symmetric planar dielectric slab waveguides
   Modal and waveguide dispersion in planar waveguides
   Step index fibers
   Numerical aperture
   Dispersion in single mode fibers
   Bit-rate, dispersion, and optical bandwidth
   Graded index optical fibers
   Light absorption and scattering
   Attenuation in optical fibers
The region of a higher refractive index \((n_1)\) is called the **core**, and the region of a lower refractive index \((n_2)\) sandwiching the core is called the **cladding**.

Our goal is to find the conditions for light rays to propagate along such waveguides.
Constructive interference occurs between A and C to achieve maximal transmitted light intensity.

\[ \Delta \phi(AC) = k_1(AB + BC) - 2\phi \]
\[ = k_1[BC \cos(2\theta) + BC] - 2\phi \]
\[ = k_1 \frac{d}{\cos \theta} \left[ 2\cos^2 \theta - 1 + 1 \right] - 2\phi \]
\[ = k_1 [2d \cos \theta] - 2\phi \]
\[ \Rightarrow k_1 [2d \cos \theta] - 2\phi = m(2\pi), \ m = 0,1,2,... \]

A and C are on the same wavefront. They must be in phase.

OR

Constructive interference occurs between A and C to achieve maximal transmitted light intensity.

\[ \Delta \phi(AC) = m(2\pi), \ m=0,1,2,... \]

Waveguide condition:

\[ \left[ \frac{2\pi n_1(2a)}{\lambda} \right] \cos \theta_m - \phi_m = m\pi \]

\( \phi_m \) is a function of \( \theta_m \).
B and B' are on the same wavefront. They must be in phase.

Constructive interference occurs between B and B' to achieve maximal transmitted light intensity.

\[
\Delta \phi = k_1(AB) - 2\phi - k_1(A'B')
\]

\[
= k_1(AB) - k_1[AB\cos(\pi - 2\theta)] - 2\phi
\]

\[
= k_1(AB)(2\cos^2 \theta) - 2\phi
\]

\[
= k_1 \frac{d}{\cos \theta} [2\cos^2 \theta] - 2\phi
\]

\[
\Rightarrow k_1[2d \cos \theta] - 2\phi = m(2\pi), m = 0,1,2,...
\]

\[
\Delta \phi(BB') = m(2\pi), m=0,1,2,...
\]

Waveguide condition:

\[
\left[ \frac{2\pi n_1(2a)}{\lambda} \right] \cos \theta_m - \phi_m = m\pi
\]
To obtain the waveguide condition and solve the propagation modes for the symmetric planar dielectric waveguides:

(1) The wave optics approach
   Solve Maxwell’s equations. There is no approximations and the results are rigorous.

(2) The coefficient matrix approach
   Straightforward. Not suitable for multilayer problems.

(3) The transmission matrix method
   Suitable for multilayer waveguides.

(4) The modified ray model method
   It is simple, but provides less information.
Propagating Constant along the Waveguide

\[
\beta_m = k_1 \sin \theta_m = \left( \frac{2\pi n_1}{\lambda} \right) \sin \theta_m
\]

\[
\kappa_m = k_1 \cos \theta_m = \left( \frac{2\pi n_1}{\lambda} \right) \cos \theta_m
\]

\[
\left[ \frac{2\pi n_1 (2a)}{\lambda} \right] \cos \theta_m - \phi_m = m \pi
\]

Larger \( m \) leads to smaller \( \theta_m \).

\( \kappa_m \): transverse propagation constant
Electric Field Patterns

\[ \Phi_m = (k_1AC - \phi_m) - k_1(A'C) \]
\[ = k_1AC - k_1AC \cos(\pi - 2\theta_m) - \phi_m \]
\[ = k_1AC(2\cos^2\theta_m) - \phi_m \]
\[ = k_1 \frac{a - y}{\cos \theta_m} \left[ 2\cos^2 \theta_m \right] - \phi_m \]
\[ = 2k_1(a - y)\cos \theta_m - \phi_m \]
\[ \left[ \frac{2\pi n_1(2a)}{\lambda} \right] \cos \theta_m - \phi_m = m\pi \]
\[ \Phi_m(y) = m\pi - \frac{y}{a} (m\pi + \phi_m) \]
Electric Field Patterns

It is a stationary wave along the $y$-direction, which travels down the waveguide along $z$. 

\[ E_1(y, z, t) = E_0 \cos(\omega t - \beta_m z + \kappa_m y + \Phi_m) \]
\[ E_2(y, z, t) = E_0 \cos(\omega t - \beta_m z - \kappa_m y) \]
\[ E(y, z, t) = E_1 + E_2 = 2E_0 \cos\left(\kappa_m y + \frac{1}{2} \Phi_m\right) \cos\left(\omega t - \beta_m z + \frac{1}{2} \Phi_m\right) \]
The lowest mode \((m = 0)\) has a maximum intensity at the center and moves along \(z\) with a propagation constant of \(\beta_0\). There is a propagating evanescent wave in the cladding near the boundary.
Electric Field Patterns

The diagram illustrates the electric field patterns in a fiber optic cable. The core has a lower index of refraction $n_1$, and the cladding has a higher index of refraction $n_2$. The electric field $E(y)$ is shown along the $y$-axis. The figure highlights three modes: $m = 0$, $m = 1$, and $m = 2$. The core radius is $2a$.
Broadening of the Output Pulse

Each light wave that satisfies the waveguide condition constitutes a mode of propagation. The integer $m$ identifies these modes and is called the mode number.

$\theta_m$ is smaller for larger $m$. Higher modes exhibit more reflections and penetrate more into the cladding.

Short-duration pulses of light transmitted through the waveguide will be broadened in terms of time duration.
Single and Multimode Waveguides

\[
\left[ \frac{2\pi n_1(2a)}{\lambda} \right] \cos \theta_m - \phi_m = m\pi
\]

\[
\theta_m \geq \theta_c \Rightarrow
\]

\[
\sin \theta_m \geq \frac{n_2}{n_1} \Rightarrow
\]

\[
1 - \cos^2 \theta_m \geq \frac{n_2^2}{n_1^2} \Rightarrow
\]

\[
\left[ \frac{m\pi + \phi_m}{4\pi a n_1 / \lambda} \right]^2 - 1 \leq -\frac{n_2^2}{n_1^2} \Rightarrow
\]

\[
\frac{m\pi + \phi_m}{4\pi a / \lambda} \leq \left( n_1^2 - n_2^2 \right)^{1/2}
\]

\[
\Rightarrow \begin{cases} 
V = \frac{2\pi a}{\lambda} \left( n_1^2 - n_2^2 \right)^{1/2} \\
\frac{m}{\pi} \leq \frac{2V - \phi_m}{\pi}
\end{cases}
\]

\[V\text{-number (}V\text{-parameter, normalized thickness, normalized frequency)}\text{ is a characteristic parameter of the waveguide.}\]

When \( V < \pi/2 \), \( m = 0 \) is the only possibility and only the fundamental mode (\( m = 0 \)) propagates along the waveguide. It is termed as the **single-mode** waveguide. \( \lambda_c \) that satisfies \( V = \pi/2 \) is the **cut-off wavelength**. Above this wavelength, only the fundamental mode will propagate.
The phase change at TIR depends on the polarization of the electric field, and it is different for $E_\perp$ and $E_\parallel$. These two fields require different angles $\theta_m$ to propagate along the waveguide.
Example: waveguide modes
Consider a planar waveguide with a core thickness 20 \( \mu \)m, \( n_1 = 1.455 \), \( n_2 = 1.440 \), light wavelength of 900 nm. Given the waveguide condition and the phase change in TIR for the TE mode, find angles \( \theta_m \) for all the modes.

\[
\left[ \frac{2\pi n_1 (2a)}{\lambda} \right] \cos \theta_m - \phi_m = m\pi
\]

\[
\tan\left( \frac{1}{2} \phi_m \right) = \frac{\left[ \sin^2 \theta_m - \left( \frac{n_2}{n_1} \right)^2 \right]^{1/2}}{\cos \theta_m}
\]

\[
\tan\left( \frac{2\pi n_1 a}{\lambda} \cos \theta_m - m \frac{\pi}{2} \right) = \frac{\left[ \sin^2 \theta_m - \left( \frac{n_2}{n_1} \right)^2 \right]^{1/2}}{\cos \theta_m}
\]
LHS (left-hand side):
\[ \tan \left( \frac{2\pi n_1 a}{\lambda} \cos \theta_m - m \frac{\pi}{2} \right) \] versus \( \theta_m \)

RHS (right-hand side):
\[ f(\theta_m) = \frac{\sin^2 \theta_m - \left( \frac{n_2}{n_1} \right)^2}{\cos \theta_m} \] versus \( \theta_m \)
Penetration depth of the evanescent wave:

\[ \delta_m = \frac{\lambda}{2\pi} \left( n_1^2 \sin^2 \theta_m - n_2^2 \right)^{-1/2} \]

<table>
<thead>
<tr>
<th>( m )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta_m )</td>
<td>89.2°</td>
<td>88.3°</td>
<td>87.5°</td>
<td>86.7°</td>
<td>85.9°</td>
<td>85.0°</td>
<td>84.2°</td>
<td>83.4°</td>
<td>82.6°</td>
<td>81.9°</td>
</tr>
<tr>
<td>( \delta_m )</td>
<td>0.691</td>
<td>0.702</td>
<td>0.722</td>
<td>0.751</td>
<td>0.793</td>
<td>0.866</td>
<td>0.970</td>
<td>1.15</td>
<td>1.57</td>
<td>3.83</td>
</tr>
</tbody>
</table>

\( \delta_m \) in \( \mu m \).

An accurate solution of the angle for the fundamental TE mode is 89.172°. The angle for the fundamental TM mode is 89.170°, which is almost identical to the angle for the TE mode.
Example: the number of modes
Estimate the number of modes that can be supported in a planar dielectric waveguide that is 100 µm wide and has \( n_1 = 1.490 \), \( n_2 = 1.470 \). The free-space light wavelength is 1 µm.

\[
m \leq \frac{2V - \phi}{\pi} \approx \frac{2V}{\pi}
\]

\[
V = \frac{2\pi a}{\lambda} \left( n_1^2 - n_2^2 \right)^{1/2}
\]

\[
= \frac{100\pi}{1} \left( 1.490^2 - 1.470^2 \right)^{1/2} = 76.44
\]

\[
m \leq \frac{2 \times 76.44}{\pi} = 48.7
\]

There are about 49 modes.

\[
M = \text{Int} \left( \frac{2V}{\pi} \right) + 1
\]


text box:

\[
M = \text{Int} \left( \frac{2V}{\pi} \right) + 1
\]

Int \((x)\) is the integer function. It removes the decimal fraction of \(x\).
**Example: mode field width (MFW),** $2w_0$

The field distribution along $y$ penetrates into the cladding. The extent of the electric field across the waveguide is therefore more than $2a$. The penetrating field is due to the evanescent wave and it decays exponentially according to

$$E_{\text{cladding}}(y') = E_{\text{cladding}}(0)\exp(-\alpha_{\text{cladding}}y')$$

The penetration depth

$$\delta_{\text{cladding}} = \frac{1}{\alpha_{\text{cladding}}} = \frac{\lambda}{2\pi} \left( n_1^2 \sin^2 \theta_i - n_2^2 \right)^{-1/2}$$

For $m = 0$ mode (axial mode), $\theta_i \to 90^\circ$

$$\delta_{\text{cladding}} \approx \frac{\lambda}{2\pi} \left( n_1^2 - n_2^2 \right)^{-1/2} = \frac{a}{V}$$

The mode field width

$$2w_0 \approx 2a + 2 \frac{a}{V} = 2a \frac{V + 1}{V}$$

For optical fibers, it is called the **mode field diameter (MFD).**
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The group velocity defines the speed with which energy or information is propagated because it defines the speed of the envelope of the amplitude variation.

To find out the group velocity, we need to know the change of $\omega$ with respect to $k$. The $\omega$ versus $k$ characteristics is called the dispersion relation or dispersion diagram.
Waveguide condition

\[
\left[ \frac{2\pi n_1 (2a)}{\lambda} \right] \cos \theta_m - \phi_m = m \pi
\]

\(\theta_m\) depends on the waveguide properties \((n_1, n_2, \text{ and } a)\) and the light frequency, \(\omega\).

\[
\beta_m = k_1 \sin \theta_m = \frac{2\pi n_1}{\lambda} \sin \theta_m
\]

The slope \(d\omega/d\beta_m\) at any frequency is the group velocity \(v_g\).

\(\omega\) is a function of \(\beta_m\) — waveguide dispersion diagram.
Intermodal Dispersion

The lowest mode \((m = 0)\) has the slowest group velocity, close to \(c/n_1\), because the lowest mode is contained mainly in the core, which has a larger refractive index \(n_1\). The highest mode has the highest group velocity, close to \(c/n_2\) because a portion of the field is carried by the cladding, which has the smaller refractive index \(n_2\). Different modes take different time to travel the length of the fiber (for an exact monochromatic light wave). This phenomenon is called \textit{intermodal dispersion}. 
A direct consequence is that a short duration light pulse signal that is coupled into the waveguide will travel along the guide via the various allowed modes with different group velocities. The reconstruction of the light pulse at the receiving end from the various modes will result in a broadened signal. The intermodal dispersion can be estimated by

$$\Delta\tau = \frac{L}{v_{g\text{min}}} - \frac{L}{v_{g\text{max}}}$$

$$v_{g\text{min}} \approx \frac{c}{n_1}, \quad v_{g\text{max}} \approx \frac{c}{n_2}$$

$$\implies \frac{\Delta\tau}{L} \approx \frac{n_1 - n_2}{c}$$

$$\Delta\tau \approx 67\text{ns/km}$$

$$n_1 = 1.48$$

$$n_2 = 1.46$$
The group velocity $d\omega/d\beta_m$ of a single mode changes with the frequency $\omega$. If the light source contains various frequencies (there is no perfect monochromatic wave), different frequencies will travel at different velocities. This is called **waveguide dispersion**.

The refractive index of a material is usually a function of the light frequency. The $n(\omega)$ dependence also results in the change in the group velocity of a given mode. This is called **material dispersion**. Waveguide dispersion and material dispersion combined together are called **intramodal dispersion**.
The transit time $\tau$ of a light pulse represents a delay between the output and the input. The signal delay time per unit distance, $\tau/L$, is called the group delay ($\tau_g$).

\[
\tau_g = \frac{\tau}{L} = \frac{1}{v_g} = \frac{d\beta}{d\omega}
\]
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Step Index Fibers

Normalized index difference \[ \Delta = \frac{n_1 - n_2}{n_1} \] for practical fibers, \( \Delta \ll 1 \)

The general ideas for guided wave propagation in planar waveguides can be extended to step indexed optical fibers with certain modifications.

The planar waveguide is bounded only in one dimension. Distinct modes are labeled with one integer, \( m \). The cylindrical fiber is bounded in two dimensions. Two integers, \( l \) and \( m \), are required to label all the possible guided modes.
There are two types of light rays for cylindrical fibers. A **meridional** ray enters the fiber through the axis and also crosses the fiber axis on each reflection as it zigzags down the fiber. A **skew** ray enters the fiber off the fiber axis and zigzags down the fiber without crossing the axis. It has a helical path around the fiber axis. Guided meridional rays are either TE or TM type. Guided skew rays can have both electric and magnetic field components along $z$, which are called hybrid modes (EH or HE modes).
We usually consider *linearly polarized* light waves that are guided in step index fibers. A guided mode along the fiber is represented by the propagation of an electric field distribution $E_{lm}(r, \varphi)$ along $z$.

$$E_{LP} = E_{lm}(r, \varphi) \exp[j(\omega t - \beta_{lm} z)]$$
V-Number of Step Index Fibers

\[ V = \frac{2\pi a}{\lambda} \left( n_1^2 - n_2^2 \right)^{1/2} \quad a \text{ is the radius of the core.} \]

\[ \Delta = \frac{(n_1 - n_2)}{n_1} \quad n_1 > n_2 \]

\[ n = \frac{(n_1 + n_2)}{2} \quad \Delta << 1 \]

\[ V = \frac{2\pi a}{\lambda} \left[ (n_1 + n_2)(n_1 - n_2) \right]^{1/2} = \frac{2\pi a}{\lambda} \left( 2n_1 n\Delta \right)^{1/2} \]

When the \( V \)-number is smaller than 2.405, only the fundamental mode (LP\(_{01}\)) can propagate through the fiber core (single-mode fiber). The cut-off wavelength \( \lambda_c \) above which the fiber becomes single-mode is given by

\[ V_{\text{cut-off}} = \frac{2\pi a}{\lambda_c} \left( n_1^2 - n_2^2 \right)^{1/2} = 2.405 \]
The number of modes $M$ in a step index fiber can be estimated by

$$M \approx \frac{V^2}{2}$$

Since the propagation constant $\beta_{lm}$ depends on the waveguide properties and wavelength, a normalized propagation constant is usually defined. This normalized propagation constant, $b$, depends only on the $V$-number.

$$b = \left( \frac{\lambda \beta_{lm}}{2\pi} \right)^2 - n_2^2$$

- $b = 0$ corresponds to $\beta_{lm} = \frac{2\pi n_2}{\lambda}$
- $b = 1$ corresponds to $\beta_{lm} = \frac{2\pi n_1}{\lambda}$
Example: A Multimode Fiber
A step index fiber has a core of refractive index 1.468 and diameter 100 $\mu$m, a cladding of refractive index of 1.447. If the source wavelength is 850 nm, calculate the number of modes that are allowed in this fiber.

\[ V = \frac{2\pi a}{\lambda} \sqrt{n_1^2 - n_2^2} = \frac{\pi 100}{0.85} \sqrt{1.468^2 - 1.447^2} = 91.44 \]

\[ M \approx \frac{V^2}{2} = \frac{91.44^2}{2} = 4181 \]

Example: A Single Mode Fiber
What should be the core radius of a single mode fiber that has a core refractive index of 1.468 and a cladding refractive index of 1.447, and is to be used for a source wavelength of 1.3 $\mu$m?

\[ V = \frac{2\pi a}{\lambda} \sqrt{n_1^2 - n_2^2} = \frac{2\pi a}{1.3} \sqrt{1.468^2 - 1.447^2} \leq 2.405 \]

\[ a \leq 2.01 \mu m \]
**Example: Single Mode Cut-Off Wavelength**

What is the cut-off wavelength for single mode operation for a fiber that has a core with a diameter of 7 μm, a refractive index of 1.458, and a cladding of refractive index of 1.452? What is the $V$-number and the mode field diameter (MFD) when operating at $\lambda = 1.3$ μm?

\[
V = \frac{2\pi a}{\lambda} \sqrt{n_1^2 - n_2^2} = \frac{\pi 7}{\lambda} \sqrt{1.458^2 - 1.452^2} \leq 2.405
\]

\[
\lambda \geq 1.208 \, \mu m
\]

When $\lambda = 1.3$ μm:

\[
V = \frac{2\pi a}{\lambda} \sqrt{n_1^2 - n_2^2} = \frac{\pi 7}{1.3} \sqrt{1.458^2 - 1.452^2} = 2.235
\]

\[
2w_0 \approx (2a) \frac{V + 1}{V} = 7 \times \frac{2.235 + 1}{2.235} = 10.13 \, \mu m
\]
Example: Group Velocity and Delay
Consider a single mode fiber with core and cladding indices of 1.448 and 1.440, core radius of 3 μm, operating at 1.5 μm. Given that we can approximate the fundamental mode normalized propagation constant by

\[ b \approx \left( 1.1428 - \frac{0.996}{V} \right)^2 \quad (1.5 < V < 2.5) \]

Calculate the propagation constant \( \beta \). Change the operating wavelength to \( \lambda' \) by a small amount, say 0.01%, and then recalculate the new propagation constant \( \beta' \). Then determine the group velocity of the fundamental mode at 1.5 μm, and the group delay \( \tau_g \) over 1 km of fiber.

\[
V = \frac{2\pi a}{\lambda} \sqrt{n_1^2 - n_2^2} \quad k = \frac{2\pi}{\lambda} \\
\]

\[
b = \frac{(\beta / k)^2 - n_2^2}{n_1^2 - n_2^2} \quad \omega = \frac{2\pi c}{\lambda}
\]
The group velocity is

\[ v_g = \frac{\omega' - \omega}{\beta' - \beta} = \frac{(1.255768 - 1.255642) \times 10^{15}}{(6.044817 - 6.044211) \times 10^6} = 2.0792 \times 10^8 \text{ m/s} \]

The group delay is

\[ \tau_g = \frac{L}{v_g} = \frac{1000}{2.0792 \times 10^8} = 4.81 \mu s \]
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Numerical Aperture

Only the rays that fall within a certain cone at the input of the fiber can propagate through the optical fiber.

Maximum acceptance angle $\alpha_{\text{max}}$ is that which just gives TIR at the core-cladding interface.

$$n_0 \sin \alpha_{\text{max}} = n_1 \sin(90^\circ - \theta_c)$$

$$\sin \theta_c = n_2 / n_1$$

$$\sin \alpha_{\text{max}} = \frac{n_1}{n_0} \cos \theta_c = \frac{(n_1^2 - n_2^2)^{1/2}}{n_0}$$

$$\text{NA} = \left(n_1^2 - n_2^2\right)^{1/2}$$

$$\sin \alpha_{\text{max}} = \frac{\text{NA}}{n_0}$$

$\alpha_{\text{max}}$: maximum acceptance angle

$2\alpha_{\text{max}}$: total acceptance angle
Example: A Multimode Fiber and Total Acceptance Angle

A step index fiber has a core diameter of 100 μm and a refractive index of 1.480. The cladding has a refractive index of 1.460. Calculate the numerical aperture of the fiber, acceptance angle from air, and the number of modes sustained when the source wavelength is 850 nm.

\[ \text{NA} = \sqrt{n_1^2 - n_2^2} = \sqrt{1.480^2 - 1.460^2} = 0.2425 \]

\[ \sin \alpha_{\max} = \frac{\text{NA}}{n_0} = \frac{0.2425}{1.0} \]

\[ \alpha_{\max} = 14^\circ \]

\[ V = \frac{2\pi a}{\lambda} \sqrt{n_1^2 - n_2^2} = \frac{2\pi a}{\lambda} \text{NA} = \frac{\pi 100}{0.85} 0.2425 = 89.63 \]

\[ M \approx \frac{V^2}{2} = \frac{89.63^2}{2} = 4017 \]
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Material Dispersion

There is no intermodal dispersion in single-mode fibers. **Material dispersion** is due to the dependence of the refractive index on the free space wavelength.

\[
\Delta \tau = |D_m| \Delta \lambda
\]

\(D_m\) is called the **material dispersion coefficient**. Dispersion is expressed as spread per unit length because slower waves fall further behind the faster waves over a longer distance.
Material Dispersion

\[ D_m \approx -\frac{\lambda}{c} \left( \frac{d^2 n}{d\lambda^2} \right) \]
Waveguide Dispersion

Waveguide dispersion is due to that the group velocity \( d\omega/d\beta \) varies as a function of \( \lambda \).

\[
\Delta \tau = \frac{|D_w|\Delta \lambda}{L}
\]

\( D_w \) is called the waveguide dispersion coefficient.

\[
b = \left( \frac{\lambda \beta_{lm}}{2\pi} \right)^2 - n_2^2
\]

\[
V = \frac{2\pi a}{\lambda} \sqrt{n_1^2 - n_2^2}
\]
Waveguide Dispersion

\[ D_w \approx \frac{1.984N_g^2}{(2\pi a)^2 2cn_2^2} \quad \text{for} \quad 1.5 < V < 2.4 \]

\[ D_m \quad \text{and} \quad D_w \quad \text{have opposite tendencies.} \]
Profile Dispersion

There is an additional dispersion mechanism called the profile dispersion that arises because the group velocity of the fundamental mode, $v_{g,01}$, also depends on the normalized index difference. $\Delta$ is dependent on the wavelength due to material dispersion characteristics, i.e., $n_1$ versus $\lambda$ and $n_2$ versus $\lambda$ behavior. Therefore, in reality, profile dispersion originates from material dispersion.

$$\Delta = \frac{n_1 - n_2}{n_1} \quad \frac{\Delta \tau}{L} = |D_p| \Delta \lambda$$

$D_p$ is called the profile dispersion coefficient.

$D_p$ is less than 1 ps km$^{-1}$ nm$^{-1}$, much smaller than $D_m$ and $D_w$. 
Chromatic Dispersion

In single-mode fibers, the dispersion of a propagating pulse arises because of the finite width $\Delta \lambda$ of the source spectrum. This type of dispersion caused by a range of source wavelengths is generally termed **chromatic dispersion**, including material, waveguide, and profile dispersion, since they are all dependent on $\Delta \lambda$.

$$
\frac{\Delta \tau}{L} = \left| D_m + D_w + D_p \right| \Delta \lambda
$$

$$
D_{ch} = D_m + D_w + D_p
$$
Polarization dispersion arises when the refractive index is not isotropic. When the refractive index depends on the direction of the electric field, the propagation constant of a given mode depends on the polarization. The anisotropic $n_1$ and $n_2$ may result from the fabrication process (changes in the glass composition, geometry, and induced local strain). Typically, polarization dispersion is less than a fraction of 1 ps km$^{-1}$. Polarization dispersion scales roughly with $L^2$. 

![Diagram of polarization dispersion](image)
Dispersion Flattened Fibers

$D_w$ can be adjusted by changing the waveguide geometry, for example, using fibers with multiple cladding layers (such fibers are more difficult to manufacture). It is often desirable to have minimum dispersion over a range of wavelengths. For example, fibers with dispersion of $1–3 \text{ ps km}^{-1} \text{ nm}^{-1}$ over the wavelength range of $1.3–1.6 \mu\text{m}$ allow for wavelength multiplexing, e.g., using a number of wavelengths as communication channels.
Example: Material, Waveguide, and Chromatic Dispersion

A single-mode fiber has a core of SiO$_2$-13.5 mol%GeO$_2$ for which the material and waveguide dispersion coefficients are shown in the figure. This fiber is excited from a 1.5 μm laser source with a linewidth $\Delta \lambda_{1/2}$ of 2 nm. What is the dispersion per km of the fiber if the core diameter $2a$ is 8 μm? What should be the core diameter for zero chromatic dispersion at $\lambda = 1.5$ μm?

At $\lambda = 1.5$ μm, $D_m = +10$ ps km$^{-1}$ nm$^{-1}$
When $a = 4$ μm, $D_w = -6$ ps km$^{-1}$ nm$^{-1}$
The chromatic dispersion coefficient is

$$D_{ch} = D_m + D_w = 10 - 6 = 4 \text{ps km}^{-1} \text{nm}^{-1}$$

The chromatic dispersion is

$$\Delta \tau_{1/2} / L = |D_{ch}| \Delta \lambda_{1/2} = (4 \text{ps km}^{-1} \text{nm}^{-1})(2 \text{nm}) = 8 \text{ps km}^{-1}$$

The chromatic dispersion will be zero at 1.5 μm when $D_w = - D_m$ and hence $D_w = -10$ ps km$^{-1}$ nm$^{-1}$. The core radius should therefore be about 3 μm. The dispersion is zero only at one wavelength.
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Modal and waveguide dispersion in planar waveguides
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✓ Bit-rate, dispersion, and optical bandwidth
Graded index optical fibers
Light absorption and scattering
Attenuation in optical fibers
In digital communications, signals are generally sent as light pulses along an optical fiber. Information is first converted to an electrical signal in the form of pulses that represent bits of information. The electrical signal drives a laser diode whose light output is coupled into a fiber for transmission. The light output at the destination end of the fiber is coupled to a photodetector that converts the light signal back to an electrical signal. The information bits are then decoded from this electrical signal. Engineers are interested in the maximum rate at which the digital data can be transmitted along the fiber. This rate is called the bit rate capacity $B$ (bits per second) of the fiber.
Suppose we feed light pulses of short duration into the fiber. The output pulses will be broadened due to various dispersion mechanisms. The dispersion is typically measured between half-power (or intensity) points and is called full width at half power (FWHP), or full width at half maximum (FWHM).

To clearly distinguish two consecutive pulses, that is no intersymbol interference, requires that they be separated from peak to peak by at least $2\Delta \tau_{1/2}$ (intuitively).
There are two types of bit rates. One is called the return-to-zero (RZ) bit rate, for which a pulse representing the binary information 1 must return to zero before the next binary information. The other is called the non-return-to-zero (NRZ) bit rate, for which two consecutive binary 1 pulses don’t have to return to zero in between, that is, the two pulses can be brought closer.

In most cases we refer to the RZ bit rate.
The maximum bit rate depends on the input pulse shape, fiber dispersion characteristics (hence the output pulse shape), and the modulation scheme of information bits. For Gaussian output light pulses $h(t)$ centered at 0:

$$h(t) = \frac{1}{\sqrt{2\pi \sigma^2}} e^{-\frac{t^2}{2\sigma^2}}$$

Standard deviation $\sigma = 0.425 \Delta \tau_{1/2}$

$$B \approx \frac{0.25}{\sigma} = \frac{0.59}{\Delta \tau_{1/2}}$$
Bit Rate and Dispersion

Dispersion increases with fiber length $L$ and also with the wavelength range of the source, $\sigma_\lambda = 0.425 \Delta \lambda_{1/2}$. It is therefore more appropriate to specify the product of the bit rate $B$ and the fiber length $L$ at the operating wavelength.

$$BL \approx \frac{0.25L}{\sigma_{\text{output}}} = \frac{0.25L}{L|D_{\text{ch}}|\sigma_{\lambda,\text{input}}} = \frac{0.25}{|D_{\text{ch}}|\sigma_{\lambda,\text{input}}}$$

$BL$ is a characteristic of the fiber, through $D_{\text{ch}}$, and also of the wavelength range of the source. In specifications, the fiber length is taken as 1 km and its unit is therefore Gb s$^{-1}$ km.

When both chromatic (intramodal) and intermodal dispersion are present and needed to be taken into account. The overall dispersion can be found according to

$$\sigma_{\text{overall}}^2 = \sigma_{\text{intermodal}}^2 + \sigma_{\text{intramodal}}^2$$
Optical and Electrical Bandwidth

The input light intensity into the fiber can be modulated to be sinusoidal. The light output intensity at the fiber destination should also be sinusoidal with a phase shift due to the time it takes for the signal to travel along the fiber. We can determine the transfer characteristics of the fiber by feeding in light intensity signals with various frequencies.
Optical and Electrical Bandwidth

The response, as defined by \( P_o/P_i \), is flat and falls with frequency when the frequency becomes too large so that dispersion effects smear out the light at the output. The frequency \( f_{op} \) at which the output intensity is 50% below the flat region defines the optical bandwidth of the fiber and hence the useful frequency range in which modulated optical signals can be transferred along the fiber. For Gaussian output light pulses, we have

\[
f_{op} \approx 0.75B \approx \frac{0.19}{\sigma}
\]

The electrical signal from the photodetector (current or voltage) is proportional to the fiber output light intensity. The electrical bandwidth, \( f_{el} \), is usually defined as the frequency at which the electrical signal is 70.7% of its low frequency value.
### Relationship between dispersion parameters, maximum bit rates, and bandwidths

<table>
<thead>
<tr>
<th>Dispersed output pulse shape</th>
<th>FWHM, $\Delta \tau_{1/2}$</th>
<th>$B$ (RZ)</th>
<th>$B'$ (NRZ)</th>
<th>$f_{\text{op}}$</th>
<th>$f_{\text{el}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaussian with standard deviation $\sigma$</td>
<td>$\sigma = 0.425\Delta \tau_{1/2}$</td>
<td>0.25/\sigma</td>
<td>0.5/\sigma</td>
<td>0.75B = 0.19/\sigma</td>
<td>0.71f_{\text{op}} = 0.13/\sigma</td>
</tr>
<tr>
<td>Rectangular with full width $\Delta T$</td>
<td>$\sigma = 0.29\Delta T = 0.29\Delta \tau_{1/2}$</td>
<td>0.25/\sigma</td>
<td>0.5/\sigma</td>
<td>0.69B = 0.17/\sigma</td>
<td>0.73f_{\text{op}} = 0.13/\sigma</td>
</tr>
</tbody>
</table>
Example: Bit Rate and Dispersion
Consider an optical fiber with a chromatic dispersion coefficient $8 \text{ ps km}^{-1} \text{ nm}^{-1}$ at an operating wavelength of $1.5 \ \mu\text{m}$. Calculate the bit rate-distance product ($BL$), and the optical and electrical bandwidths for a 10 km fiber if a laser diode source with a FWHP linewidth $\Delta\lambda_{1/2}$ of 2 nm is used.

The FWHP dispersion at the output side is

$$\Delta \tau_{1/2} / L = |D_{ch}| \Delta \lambda_{1/2} = \left(8 \text{ ps km}^{-1} \text{ nm}^{-1}\right)(2 \text{ nm}) = 16 \text{ ps km}^{-1}$$

Assume a Gaussian light pulse shape, the RZ bit rate-distance product is

$$BL = \frac{0.25L}{\sigma} = \frac{0.25L}{0.425\Delta \tau_{1/2}} = \frac{0.25}{0.425 \times 16} = 36.8 \text{ Gbps}^{-1}\text{km}$$

The optical and electrical bandwidths for a 10 km distance are

$$f_{op} = 0.19 / \sigma = 0.19 / (0.425\Delta \tau_{1/2}) = 0.19 / (0.425 \times 16 \times 10) = 2.8 \text{ GHz}$$

$$f_{el} = 0.71f_{op} = 0.71 \times 2.8 = 2.0 \text{ GHz}$$
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✓ Graded index optical fibers
Light absorption and scattering
Attenuation in optical fibers
Single mode step index fibers have small NA and the amount of light coupled into a fiber is limited. Multimode step index fibers have large NA and core diameters, which allow for more light power launched into a fiber. However, they suffer from intermodal dispersion. Intuitively, those rays that experience less reflections will arrive at the end of the fiber earlier.
In the **graded index (GRIN)** fiber, the refractive index is not constant within the core but decreases from \( n_1 \) at the center, as a power law, to \( n_2 \) at the cladding. Such a refractive index profile is capable of minimizing intermodal dispersion. **Intuitively**, the velocity, \( c/n \), is not constant and increases away from the center. A ray such as 2 that has a longer path than ray 1 experiences a larger velocity during a part of its journey to enable it to catch up with ray 1. Similarly, ray 3 experiences a larger velocity than ray 2 during part of its propagation to catch up with ray 2.
Graded Index (GRIN) Fibers

The refractive index profile can be described by a power law with an index $\gamma$, which is called the \textbf{profile index} (or the \textbf{coefficient of index grating}).

\[
\begin{align*}
   n &= n_1 \left[1 - 2\Delta \left(\frac{r}{a}\right)^\gamma\right]^{1/2} & r < a \\
   n &= n_2 & r = a
\end{align*}
\]

The intermodal dispersion is minimized when ($\Delta$ is small)

\[
\gamma = \frac{4 + 2\Delta}{2 + 3\Delta} = \frac{2 + \Delta}{1 + \frac{3}{2}\Delta} \approx \left(2 + \Delta\right)\left(1 - \frac{3}{2}\Delta\right) \approx 2\left(1 - \Delta\right)
\]

With the optimal profile index, the dispersion in the output light pulse per unit length is given by

\[
\frac{\sigma_{\text{intermodal}}}{L} \approx \frac{n_1}{20\sqrt{3}c} \Delta^2
\]
<table>
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✓ Light absorption and scattering
Attenuation in optical fibers
Absorption

In absorption, some of the energy from the propagating wave is converted to other forms of energy, for example, to heat by the generation of lattice vibrations.

In general, light propagating through a material becomes *attenuated* in the direction of propagation.
When a propagating wave encounters a small dielectric particle or a small inhomogeneous region whose refractive index is different from the average refractive index of the medium, the field forces dipole oscillations in the dielectric particle or region by polarizing it, leading to the emission of electromagnetic waves in many directions so that a portion of the light energy is directed away from the incident beam.
Whenever the size of a scattering region, whether an inhomogeneity or a small particle, is much smaller ($\lambda/10$) than the wavelength of the incident wave, the scattering process is generally termed **Rayleigh scattering**.

Lord Rayleigh, an English physicist (1877 – 1919) and a Nobel laureate (1904), made a number of contributions to wave physics of sound and optics.
- Why is the sky blue?
- Why does the sun look yellow if we look at the sun directly?
- Why does the sky around the sun appear red during sunrise and sunset?
Rayleigh scattering becomes more severe as the frequency of light increases (the wavelength decreases). Blue light that has a shorter wavelength than red light is scattered more strongly by particles in air.

When we look at the sun directly, it appears yellow because the blue light has been scattered.

When we look at the sky in any direction but the sun, our eyes receive scattered light, which appears blue.

At sunrise and sunset, the rays from the sun have to travel the longest distance through the atmosphere and have the most blue light scattered, which gives the sky around the sun its red color at these times.
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✓ Attenuation in optical fibers
Attenuation in Optical Fibers

Assume a fiber of length $L$. The input optical power is $P_{\text{in}}$. The optical power is attenuated to $P_{\text{out}}$ at the end of the fiber. We define an attenuation coefficient $\alpha$ for the fiber.

$$dP = -\alpha P \, dx$$

$$\frac{dP}{P} = -\alpha dx$$

$$\alpha = \frac{1}{L} \ln \left( \frac{P_{\text{in}}}{P_{\text{out}}} \right)$$

$$P_{\text{out}} = P_{\text{in}} \exp(-\alpha L)$$

Optical power attenuation in optical fibers is generally expressed in terms of **decibels** per unit length of fiber, typically as dB per km.

$$\alpha_{\text{dB}} = \frac{1}{L} \ln \left( \frac{P_{\text{in}}}{P_{\text{out}}} \right)$$

$$\alpha_{\text{dB}} = \frac{10}{\ln(10)} \alpha = 4.34 \alpha$$
- The sharp increase in attenuation at wavelengths beyond 1.6 μm is due to the energy absorption by the lattice vibrations of silica.
- Two peaks at 1.4 μm and 1.24 μm are due to OH⁻ ions in silica glass.
- The overall background is due to Rayleigh scattering because of the amorphous structure of silica glass (impossible to eliminate Rayleigh scattering in glass).
The attenuation $\alpha_R$ in a single component glass due to Rayleigh scattering is approximately given by

$$\alpha_R \approx \frac{8\pi^3}{3\lambda^4} \left( n^2 - 1 \right)^2 \beta_T k_B T_f$$

$\lambda$ is the free space wavelength. $T_f$ is called the fictive temperature, at which the liquid structure during the cooling of the fiber is frozen to become the glass structure. $\beta_T$ is the isothermal compressibility of the glass at $T_f$. 

**Attenuation in Optical Fibers**
Microbending and Macrobending Losses

**Microbending** is due to a sharp local bending of the fiber that changes the guide geometry and refractive index profile locally, which leads to some of the light energy radiating away from the guiding direction.

Local bending leads to an decrease in the incidence angle, which induces either an increase in the penetration depth into the cladding or a loss of total internal reflection.
Microbending and Macrobending Losses

Microbending loss increases sharply with decreasing radius of curvature.

Measured microbending loss for a 10-cm fiber bent with different amounts of radius of curvature. Single mode fiber with a core diameter of 3.9 μm, cladding radius 48 μm, Δ = 0.00275, NA ≈ 0.11, V ≈ 1.67 and 2.08.
Microbending and Macrobending Losses

Macrobending loss is due to small changes in the refractive index of the fiber due to induced strain when it is bent during its use, for example, when it is cabled and laid. Typically, when the radius of curvature is close to a few centimeters, macrobending loss crosses over into the regime of microbending loss.
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Photonic Crystal Fibers (PCFs)

Hollow core PCFs
- Zero dispersion
- Near-Gaussian profile
- Nearly immune to bend loss
- No reflection at the end face
- Sensing
- Ultrafast spectroscopy and switching

Nonlinear PCFs
- Expensive

https://www.thorlabs.com/
How Small A Fiber Can Be?

An electric field pattern $E_{lm}(r, \varphi)$ propagates along the fiber.

$$E_{LP} = E_{lm}(r, \varphi) \exp[j(\omega t - \beta_{lm}z)]$$
How Small A Fiber Can Be?

Due to the presence of the evanescent wave in the cladding, not all of the optical power propagating along the fiber is confined inside the core. The extent to which a propagating mode is confined to the fiber core can be measured by the ratio of the power carried in the cladding to the total power that propagates in the mode.

\[
V = \frac{P_{\text{cladding}}}{P_{\text{core}} + P_{\text{cladding}}}
\]

\[
V = \frac{2\pi a}{\lambda} (n_1^2 - n_2^2)^{1/2}
\]

\[V_{\text{cut-off}} \text{ (single mode)} = 2.405\]
How Small A Fiber Can Be?
Consider a step index fiber with a silica core ($n_1 = 1.45$). The cladding of this fiber is simply air ($n_2 = 1.0$). The laser light source is from an argon ion laser with $\lambda = 514.5$ nm (green light). If the radius of this fiber is equal to the light wavelength ($a = 514.5$ nm), then

$$V = \frac{2\pi a}{\lambda} \left( n_1^2 - n_2^2 \right)^{1/2} = 2\pi \sqrt{1.45^2 - 1.0^2} = 6.6 \quad \nu = 0.02$$

If the diameter of this fiber is equal to the light wavelength ($2a = 514.5$ nm), then

$$V = 3.3 \quad \nu = 0.1$$

If the diameter of this fiber is equal to half the light wavelength ($4a = 514.5$ nm), then

$$V = 1.6 \quad \nu = 0.5$$
For such small fibers, difficulties lie in fabrication, precise positioning, and light coupling. Propagation loss might not be a problem if our goal is to fabricate high-density integrated photonic circuits.

SnO$_2$ nanoribbons
- rectangular
- 350 nm wide
- 245 nm thick
- $\lambda_0$~525 nm

Optical Interconnects between Electronic Microprocessors
Size Incompatibility between Electronic and Photonic Circuits

There has been great interest in the use of optical interconnects to exchange digital information between electronic microprocessors. One severe limit on the integration of optical and electronic circuits is their respective sizes. Electronic circuits can be fabricated with sizes below 100 nm, while the minimum sizes of optical structures are limited by optical diffraction to the order of 1000 nm.
Plasmonic Waveguides

Bridge electronics and photonics


Plasmonic behavior is a physical concept that describes the collective oscillation of conduction electrons in a metal. Many metals can be treated as free-electron systems whose electronic and optical properties are determined by the conduction electrons. In the Drude–Lorentz model, such a metal is denoted as a plasma, because it contains equal numbers of fixed positive ions and nearly free conduction electrons. Under the irradiation of an EM wave, the free electrons are driven by the electric field to coherently oscillate at a plasma frequency of $\omega_p$ relative to the lattice of positive ions. For a bulk 3D metal with infinite sizes, $\omega_p$ is related to the number density of electrons. Quantized plasma oscillations are called plasmon resonance.

$$\omega_p = \left( \frac{Ne^2}{\varepsilon_0 m_e} \right)^{1/2}$$

<table>
<thead>
<tr>
<th>Metal</th>
<th>Theoretical $\lambda_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Na</td>
<td>200 nm</td>
</tr>
<tr>
<td>Al</td>
<td>77 nm</td>
</tr>
<tr>
<td>Ag</td>
<td>140 nm</td>
</tr>
<tr>
<td>Au</td>
<td>140 nm</td>
</tr>
</tbody>
</table>
Propagating Surface Plasmon Resonance (PSPR)

In reality, metallic structures are of finite dimensions and are surrounded by dielectric materials. Since an EM wave impinging on a metal surface only has a certain penetration depth, just the electrons on the surface are the most important. Their collective oscillations are properly termed \textit{surface plasmon polaritons} (SPPs), also often called surface plasmon resonance (SPR). For a metal–vacuum interface, the boundary condition leads to a SPR mode of $\frac{\omega_p}{2^{1/2}}$ in frequency. Such a SPR mode represents a longitudinal surface charge density wave that can travel along the surface. For this reason, this SPR is also widely known as \textit{propagating SPR} (or PSPR).
PSPR has a combined EM wave and surface charge character. There is an enhanced field component perpendicular to the interface and decaying exponentially away from the interface. This evanescent wave reflects the bound, non-radiative nature of surface plasmon resonance and prevents power from propagating away from the interface.
Decay Lengths of PSPR

The decay length, $\delta_d$, in the dielectric medium above the metal, typically air or glass, is of the order of half the wavelength of involved light.

The decay length in the metal, $\delta_m$, is typically between one and two orders of magnitude smaller than the wavelength involved, which highlights the need for good control of fabrication of plasmon-based devices at the nanometer scale.
Propagation Length of PSPR

$$\delta_{SP} = \frac{c}{\omega} \left( \frac{\varepsilon_m + \varepsilon_d}{\varepsilon_m \varepsilon_d} \right)^{\frac{3}{2}} \left( \frac{\varepsilon_m}{\varepsilon_m''} \right)^2$$

$\varepsilon_d$ is the dielectric constant of the dielectric material.
$\varepsilon_m = \varepsilon'_m + \varepsilon''_m$ is the dielectric function of the metal.

Silver has the lowest absorption losses (smallest $\varepsilon''_m$) in the visible spectrum. The propagation length for Ag is in the range of 10–100 $\mu$m, and increases to 1 mm as the wavelength moves into the 1.5 $\mu$m telecommunication band.

In the past, absorption by metals was seen as such a severe problem that surface plasmon resonance were not considered as viable for photonic elements. This view is now changing due primarily to recent demonstrations of plasmon-based components that are smaller than the propagation length. Such developments open the way to integrate plasmonic devices into circuits before propagation losses become too significant.
These characteristic length scales are important for PSPR-based photonics in addition to the associated light wavelength. The propagation length sets the upper size limit for any photonic circuit based on PSPR. The decay length in the dielectric material, $\delta_d$, dictates the maximum heights of any individual feature that might be used to control surface plasmons. The decay length in the metal, $\delta_m$, determines the minimum feature size that can be used.
The surface plasmon resonance is bound to and propagate along the bottom of V-shaped grooves milled in gold films.

\[ \lambda = 1600 \text{ nm} \]
\[ d = 1.1-1.3 \text{ \(\mu\)m} \]
\[ 2\theta = \sim 25^\circ \]

Y-splitter
Mach-Zehnder interferometer

Plasmonic Waveguides —— Grooves in Metal Films

Waveguide-ring resonator

O. Tsilipakos et al.,
http://photonics.ee.auth.gr/index.html
By changing the phase difference between the two inputs I1 and I2, the light intensities at the two outputs O1 and O2 can be controlled. The red arrows represent the optimized polarization directions for light input. The device is made of chemically synthesized Ag nanowires.

Reading Materials