Valuation of Corporate Bonds with Stochastic Default Barriers

C. H. Hui
Banking Policy Department
Hong Kong Monetary Authority
30th Floor, 3, Garden Road, Hong Kong, China
Email: Cho-Hoi_Hui@hkma.gov.hk

C. F. Lo and H. C. Lee
Physics Department
The Chinese University of Hong Kong
Shatin, Hong Kong, China
E-mail: cflo@phy.cuhk.edu.hk

Abstract
This paper develops a three-factor corporate bond valuation model that incorporates a stochastic default barrier. The default barrier is considered as the bond issuer’s liability. The model is in line with the packing-order theory of firms’ capital structures. A corporate bond defaults when the bond issuer’s leverage ratio increases above a predefined level upon default. The payoff to the bondholders in case of default is a constant fraction of the value of a default-free security with the same corporate bond face value. A closed-form solution of the corporate bond price is derived to obtain credit spreads. The three-factor model is capable of producing term structures of credit spreads that are consistent with some empirical findings.

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1. Introduction

Under the structural model approach to pricing corporate bonds, default risk is
treated as a put option on the firm (the bond issuer) asset value\(^1\). Different
default-triggering mechanisms have been considered in previous research. Black and Scholes
(1973) and Merton (1974) have been the pioneers in this approach and assumed that
default occurs only at bond maturity when the bond issuer’s asset value is less than its
liabilities due to the bond. To cope with the possibility of early default before bond
maturity, Black and Cox (1976) assume a bankruptcy-triggering level for the firm
asset value following a constant risk-free interest rate whereby default can occur at
any time. This trigger level is introduced by considering a safety covenant that
protects bondholders.

Longstaff and Schwartz (1995) (hereafter LS) extend Black and Cox’s model
to open a new research agenda in the sense that the evolution of the firm's capital
structure is no longer tied up with the payoffs of any individual claim on the firm's
assets. They assume default if the value of the firm assets falls below a fixed default
barrier. The default barrier is considered as the firm’s liability. Based on the
contribution of Black and Cox (1976) and LS, Briys and de Varenne (1997) and
Schöbel (1999) propose a default barrier following stochastic interest rate with the
Ornstein-Uhlenbeck process. This barrier is assumed to grow with time together with
the value of the firm and the expected level of leverage of the firm is therefore kept at
constant. Hui et al. (2003) develop a corporate bond valuation model that
incorporates a dynamic default barrier which is governed by stochastic interest rates

\(^1\) The second approach is the reduced-form models in which default time is a stopping time of some
given hazard rate process and the payoff upon default is specified exogenously. This approach has
and the variance of the corporate bond value. In all these models, the dynamics of the default barrier reflects the expected level of leverage of a bond issuer and its behaviour in respect to its capital structure.

Theoretical literature on corporate finance presents two theories of firms’ capital structures, which are the trade-off and pecking-order theories. The trade-off theory proposed by Bradley et al. (1984) and Harris and Raviv (1991) suggests that particular classes of firms have specific optimal liability-to-asset ratios, determined by balancing the present value of expected marginal benefits of leverage against the present value of expected marginal costs of leverage. According to the trade-off theory, each firm borrows in order to gradually move towards its optimal leverage ratio. This calls for a stationary-leverage-ratio model for pricing corporate bonds, which has been studied by Collin-Dufresne and Goldstein (2001). The stationary-leverage-ratio model considers a mean-reverting liability (i.e. a default barrier) which is associated with the dynamics of risk-free interest rates. Default of a firm occurs when the stochastic firm value hits the default barrier. Collin-Dufresne and Goldstein (2001) conclude that accounting for a bond issuer’s ability to control its level of outstanding debt in the model has a significant impact on credit spread predictions. It helps reconcile some predictions of credit spreads with empirical observations. These include credit spreads that are larger for low-leveraged firms and less sensitive to changes in firm value.

The pecking-order theory proposed by Myers and Majluf (1984) shows that, since investors are less informed about a firm’s value than existing shareholders are, external financing may be mis-priced by the market. Thus the pecking-order theory

been considered by Artzner and Delbaen (1992), Madan and Unal (1993), Jarrow, Lando, and Turnbull
suggests that firms adhere to a financial hierarchy in order to maximise their values: (i) internal funds are given preference over external ones in financing investments; (ii) if external financing is needed, firms first seek debt funding; (iii) equity is only issued as a last resort. Unlike the trade-off theory, therefore, the pecking-order theory does not envisage the existence of a target leverage ratio.

Empirical findings suggest that both the trade-off and pecking-order theories contribute towards explaining the financial behaviour of firms. Shyam-Sunder and Myers (1999) find that both theories are an excellent first-order descriptor of corporate financing behaviour, at least for their sample of mature corporations. Similarly, Bontempi and Golinelli (2001) confirm that the best way of defining the financial behaviour of Italian manufacturing firms is to formulate an explanation where both theories are admissible. Frank and Goyal (2003) show that large firm exhibit some aspects of pecking-order behaviour.

This paper uses the structural model approach to present a three-factor model of valuation of corporate bonds, which is in line with the pecking-order theory. In the model, the default barrier of a firm follows a standard Wiener process (without any mean reverse) and is correlated with the firm’s stochastic asset value and risk-free interest rates. The relationship between the asset value and the default barrier in the three-factor model is specified through their correlation. When the default barrier is considered as the firm’s liability, positive correlation implies that a firm’s future leverage ratio would be close to the current level as the movements of the values of its asset and liability are correlated. On the other hand, negative correlation would make the firm’s future leverage level deviate from the current level. This implies that the
firm’s liability value would not align with its firm value and its default risk would be increased. The correlation can be observed from the historical behaviour of the firm in regard to its leverage level. With appropriate model parameters such as negative correlation between the liability value and interest rates, the credit spread could be a decreasing function of interest rates in the three-factor model. This is consistent with the empirical findings of LS and Duffee (1998, 1999). This can be explained by a reason that firms reduce their demand for additional debt as interest rates increase.

The inclusion of the stochastic default barrier in the model is consistent with the findings by Pan (2001). Pan (2001) concludes that the inclusion of the volatility of the default barrier is important for pricing credit risk after examining the JP Morgan Chase closing par spreads of credit default swaps of 39 firms from March to November 2000 based on a simple model using uncorrelated variables of firm’s asset values and default barriers with lognormal distributions.

The source of randomness of the default barrier in the three-factor model is not solely tied to the interest rate process, that is assumed in the models developed by Briys and de Varenne (1997) and Schöbel (1999). The dynamics of the default barrier is also different from that proposed by Saà-Requejo and Santa-Clara (1999). While Saà-Requejo and Santa-Clara argue that their default barrier is stochastic, the volatility of return of the changes in the default barrier is entirely due to interest rate changes and the firm value changes. Their proposed default barrier therefore does not have its own randomness.

The LS model incorporating fixed default barriers that does not envisage the existence of a target leverage ratio is consistent with the pecking-order theory. The model cannot however generate credit spreads which replicate empirically observed
spreads. The credit spread analysis based on the three-factor model show that the use of stochastic default barriers is capable of producing term structures of credit spreads that are consistent with some empirical findings.

The dynamics of the short-term interest rate in the three-factor model is assumed to follow the Ornstein-Uhlenbeck process, i.e. the Vasicek model (Vasicek, 1977). When the leverage ratio touches the default barrier, default occurs and bondholders receive an exogenously specified number of risk-free discount bonds. We derive a closed-form solution of the bond price as a function of the firm value, default barrier and interest rates explicitly. The model parameters such as, volatility, correlation, drift and mean-level of interest rates are time dependent in the derivation.

The remainder of the paper is organised as follows: Section 2 gives an outline of the three-factor model and presents the closed-form solution of the corporate discount bond price. In Section 3, we study the credit spread term structures generated from the three-factor model with the LS model. Section 4 summarises the findings.

2. Valuation model of corporate bonds

A continuous-time framework is used to price a corporate discount bond. A firm’s asset value and liability (default barrier), and the short-term interest rate are stochastic variables. The firm value and the default barrier are assumed to follow a lognormal diffusion process. The dynamics of the interest rate is drawn from the term structure model of Vasicek (1977), i.e. the Ornstein-Uhlenbeck process. Although this assumed process is consistent with many of the observed properties of interest rates, it can allow negative interest rates. However, this assumption may still be
justifiable in the context of the valuation because given that the current value of interest rate and the mean-level are both positive, the dynamics always imply positive expected future interest rate.

Let \( V \) denote the firm value, \( Q \) denote the default barrier and \( r \) denote the short-term interest rate. Their continuous stochastic movements are modelled by the following stochastic differential equations:

\[
\frac{dV}{V} = \mu_V(t)dt + \sigma_V(t)dZ_V
\]

\[
\frac{dQ}{Q} = \mu_Q(t)dt + \sigma_Q(t)dZ_Q
\]

\[
dr = \kappa(t)(\theta(t) - r)dt + \sigma_r dZ_r
\]

where \( \sigma_Q(t) \) and \( \sigma_V(t) \) are the respective volatility values, \( \mu_Q(t) \) and \( \mu_V(t) \) are the respective drift rates, and they are time dependent. The interest rate uncertainty is driven by a Vasicek representation with the instantaneous volatility \( \sigma_r(t) \). The short-term interest rate \( r \) is mean-reverting to long-run mean \( \theta(t) \) at speed \( \kappa(t) \). The Wiener processes \( dZ_Q, dZ_V \) and \( dZ_r \) are correlated with

\[
dZ_Q dZ_V = \rho_{QV}(t)dt, \quad dZ_V dZ_r = \rho_{VR}(t)dt, \quad dZ_Q dZ_r = \rho_{QR}(t)dt.
\]

The price of the corporate discount bond is \( P(Q, V, r, t) \). Using Ito’s lemma and the standard risk neutral arguments, the partial differential equation governing the corporate discount bond is

\[
\frac{\partial P}{\partial t} = \frac{1}{2}\sigma_Q^2(t)Q^2 \frac{\partial^2 P}{\partial Q^2} + \frac{1}{2}\sigma_V^2(t)V^2 \frac{\partial^2 P}{\partial V^2} + \frac{1}{2}\sigma_r^2(t) \frac{\partial^2 P}{\partial r^2} + \rho_{QV}(t)\sigma_Q(t)\sigma_V(t)V^2 \frac{\partial^2 P}{\partial Q \partial V} + \rho_{Qr}(t)\sigma_Q(t)\sigma_r(t)Q \frac{\partial^2 P}{\partial Q \partial r} + \\
\rho_{Vr}(t)\sigma_V(t)\sigma_r(t)V \frac{\partial^2 P}{\partial V \partial r} + rQ \frac{\partial P}{\partial Q} + rV \frac{\partial P}{\partial V} + \kappa(t)(\theta(t) - r)\frac{\partial P}{\partial r} - rP
\]
The bond value is obtained by solving equation (4) subject to the final payoff condition and the boundary condition. When the firm’s liability-to-asset ratio is above a predefined level $L$, bankruptcy occurs before bond maturity at $t = 0$. This is consistent with the event of bankruptcy being associated with high levels of debt relative to the market value of the firm’s assets. The payoffs to bondholders at default are specified by

$$P(Q/V = L, r, t) = wFB(r, t) \quad t > 0; w < 1,$$

(5)

where $B(r, t)$ is the default-free bond function according to the Vasicek model, $w$ is the recovery ratio and $F$ is the bond face value. When $w = 0$, bondholders receive nothing in a financial reorganisation; and when $w > 0$, bondholders receive new securities in exchange for their original claims.

On the other hand, if the liability-to-asset ratio has never breached the predefined level $L$, the payoff to bondholders at bond maturity is

$$P(Q, V, r, t = 0) = F.$$

(6)

As shown in equation (A.12) in the Appendix, the solution of equation (4) subject to the boundary condition and the final payoff condition of equations (5) and (6) can be approximated by

$$P(Q, V, r, t) = FB(r, t) - (1 - w)FB(r, t)D(Q, V, r, t)$$

(7)

where
\[
D(Q, V, r, t) = 1 + \exp \left\{ \beta \left[ \beta C_1(t) - \ln \left( \frac{LV}{Q} \right) - C_2(t) \right] \right\} \cdot \frac{2\beta C_1(t) - \ln \left( \frac{LV}{Q} \right) - C_2(t)}{\sqrt{2C_1(t)}}
\]

\[
= \frac{\ln \left( \frac{LV}{Q} \right) + C_2(t)}{\sqrt{2C_1(t)}}
\]

\(N\) is the cumulative normal distribution function, \(\beta\) is a real number parameter, and \(C_1(t)\) and \(C_2(t)\) are defined as follows:

\[
C_1(t) = \frac{1}{2} \int_0^t \sigma_1^2(t') dt',
\]

\[
C_2(t) = \int_0^t \gamma(t') dt',
\]

\[
\sigma_1(t) = \sqrt{\sigma_0^2(t) + \sigma_v^2(t) - 2\rho_{ov}(t)\sigma_0(t)\sigma_v(t)}
\]

\[
\gamma(t) = \alpha_i(t) + \rho_{iv}(t)\sigma_1(t)\sigma_v(t)A_3(t)\exp[A_4(t)] - \frac{1}{2}\sigma_1^2(t),
\]

\[
\alpha_i(t) = \sigma_v^2(t) - \rho_{ov}(t)\sigma_0(t)\sigma_v(t),
\]

\[
\rho_{iv}(t) = \frac{\rho_{ov}(t)\sigma_0(t)\sigma_v(t)}{\sigma_i},
\]

\[
A_4(t) = -\int_0^t \kappa_r(t') dt',
\]

\[
A_3(t) = -\int_0^t \exp[-A_2(t')] dt'.
\]

The parameter \(\beta\) is generally adjusted such that the approximate solution in equation (7) provides the best approximation to the exact result (see the Appendix for the details). As the three-factor model preserves the value additivity feature of the LS
model, corporate coupon bonds can be valued simply as a portfolio of corporate
discount bonds.

Equation (7) shows that the value of a corporate discount bond depends on $V$
and $Q$ only through their ratio $Q/V$ (i.e. leverage ratio). Thus, the leverage ratio
provides a summary measure of default risk of the firm and can be viewed as a proxy
variable for the default risk of the firm. This closed-form expression of equation (7)
has an intuitive structure. The first term represents the value that the bond would have
if it were risk free. The second term represents a discount for the default risk of the
bond. The discount for default risk consists of two components. The first component,
$(1 - w)FB(r, t)$, is the present value of the loss on the bond in the event of default.
The second component, $D(Q, V, r, t)$, is the probability of default under the risk-
neutral measure. It is noted that the probability of default is independent of the face
value of the bond $F$. It is because the payoff condition specified in equation (6)
assumes that the bond issuer is capable of paying or refinancing the principle of the
bond as long as its firm value has never breached the default barrier.

Since $Q/V$ is a sufficient measure for default risk in this model, we do not need
to specify the condition of the pattern of cash payments to be made prior to the
maturity date of a bond in order to value the bond. This means that we assume that
financial distress triggers the default of all of the firm’s debt. The price of a corporate
bond is an increasing function of the default-risk measure $Q/V$. This is intuitive since
the higher the value of $Q/V$, the closer the firm is to the predefined level upon default
and the higher for default risk.
3. Credit spread analysis

The credit spread $C_s$ is defined as the difference between the yield of a corporate discount bond and the yield of an equivalent risk-free discount bond. The credit spread is thus given by the following expression:

$$C_s = -\frac{1}{t} \ln \left( \frac{P(Q,V,r,t)}{F} \right) + \frac{1}{t} \ln B(r,t)$$  \hspace{1cm} (8)

Using the closed-form solution of the discount bond value $P(Q, V, r, t)$ in equation (7), the credit spreads can be readily obtained. In the numerical calculations below, the three-factor model credit spreads are compared with the LS model credit spreads.

The common basic model parameters of the two models are as follows: $V = 1$, $\sigma_V = 0.2$, $\rho_{Vr} = 0.25$, $r = 6\%$, $\sigma_r = 0.0316$, $\theta = 6\%$, $\kappa = 1$ and $w = 0.42$. The value of $w$ is consistent with the expected recovery rate for senior unsecured debt according to Carty and Lieberman (1998). For the three-factor model, we put $\sigma_Q = 0.1$, $\rho_{QV} = -0.3$ and $\rho_{Qr} = 0.0$. The predefined level $L$ of the liability-to-asset ratio upon default is assumed to be 81% because this is the average liability-to-asset ratio for defaulted firms in the sample of Frank and Torous (1994). The parameter $\beta$ is adjusted such that the default boundary $X_1(t)$ of the liability-to-asset ratio in equation (A.10) of the Appendix is approximately equal to constant $L = Q_0/V_0$ over time. The movement of $X_1(t)$ over time is plotted in Figure 1, which shows that the approximation is reasonably good. For the LS model, a fixed default barrier is simulated by imposing zero volatility on $Q$.

Figure 2 shows the credit spread term structures of discount bonds issued by a medium-leveraged firm with $Q = 0.33$ and $\sigma_Q = 0.1$, 0.2, and 0.3. The value $Q/V = 0.33$ is similar to the liability-to-asset ratio of firms whose senior debt rating from
Standard and Poor’s is BBB (see Standard and Poor’s (2001)). The credit spreads generated from the three-factor model are higher than those generated by the LS model. This is due to the dynamics of the default barrier $Q$ that increases the default risk of the bond. The depicted term structures of the three-factor model exhibit upward slopes, which increase with $\sigma_Q$, at short maturities. Regarding the term structure based on $\sigma_Q = 0.3$, the prediction by the proposed model that credit spreads are non-zero at short maturities of two to three years. This agrees with the findings of Fama (1986) from commercial paper credit spreads. While the problem of downward-biased credit spreads at short maturities is common to all contingent-claims pricing models which assume continuous dynamics, the use of high volatility of the default barrier may rectify this problem to a certain extent. The high volatility of the default barrier may represent uncertainty about a bond issuer’s actual amount of short-term debt due to incomplete (e.g. noisy or delayed) disclosed accounting information of the issuer for investors or potential liquidity problem faced by the issuer to refinance the bond. Factors like these would increase default probability near bond maturity.

At longer maturities, the term structures display different degrees of downward-sloping depending on the values of $\sigma_Q$, where the term structure with $\sigma_Q = 0.1$ exhibits a gentle downward slope at longer maturities. Their shapes are consistent with the empirical findings by Sarig and Warga (1989) that the term structure is humped for medium-leveraged firms. While the credit spreads generally increase with $\sigma_Q$, the differences among them based on different $\sigma_Q$ are not significant at longer maturities, particularly those beyond 15 years. This reflects that the sensitivity of default risk of a medium-leveraged firm to $\sigma_Q$ reduces over time as the firm survives.
It is a question whether the differences in the term structures of the three-factor and LS models can be eliminated by simply adjusting the volatility $\sigma_V$ of the firm value $V$. The term structures of the LS model with $\sigma_V = 0.2$, 0.23 and 0.26 are therefore plotted in Figure 3 and are compared with that generated by the three-factor model with $\sigma_V = 0.2$, $\sigma_Q = 0.1$ and $\rho_QV = 0.0$. The results show that the term structures generated from the two models do not converge by just adjusting $\sigma_V$ in the LS model. While the credit spreads of the models come close to each other with time to maturity up to five years using $\sigma_V = 0.26$ in the LS model, the credit spreads of the three-factor model are higher than those of the LS model at maturities over five years. This shows that the stochastic default barrier can change credit spreads of corporate bonds in terms of both magnitude and structure.

Figure 4 shows the term structures of discount bonds issued by a low-leveraged firm with $Q = 0.12$ and $\sigma_Q = 0.1$, 0.2, and 0.3. The firm with the liability-to-asset ratio of 0.12 can be considered as an AA-rated firm. Similar to Figure 2, the credit spreads generated from the three-factor model in Figure 4 are higher than those generated by the LS model. Also, the credit spreads generally increase with $\sigma_Q$ at all maturities. In Figure 4, the shapes of the term structures based on the three-factor model are quite different from that generated by the LS model. The term structures exhibit upward slopes at short maturities. At longer maturities, the shapes of the two term structures with $\sigma_Q = 0.1$ and 0.2 continue to exhibit upward slopes at longer maturities. Both the term structures are similar to the empirical studies in Sarig and Warga (1989) and Fons (1994). Sarig and Warga (1989) find that the term structure of credit spreads is upward sloping for low-leveraged firms using pure discount bonds. Fons (1994) find that bonds rated Aa (Moody’s rating) exhibit a significant positive
relation between spreads and maturity based on the market data of Aa-rated bonds. The shape of the term structure with $\sigma_Q = 0.3$ however exhibits flat at longer maturities. This shows that the use of low volatility $\sigma_Q$ for the default barrier of a low-leveraged firm generates the term structures which are consistent with the empirical findings.

Figure 5 shows the effect of the correlation $\rho_{QV}$ between $Q$ and $V$ on the term structures of discount bonds issued by a medium-leveraged firm with $Q = 0.33$ and $\sigma_Q = 0.1$. The results show that credit spreads are sensitive to $\rho_{QV}$. The credit spreads increase when $Q$ and $V$ are more negative correlated. This is due to the reason that the opposite movements of $Q$ and $V$ caused by the negative correlation probably increase the liability-to-asset ratio of the firm so as to increase the default risk. When $Q$ and $V$ are positive correlated, the expected liability-to-asset ratio would be more stable over time and thus the default risk is reduced. Figure 5 also illustrates that different $\rho_{QV}$ can produce quite diverse shapes of the term structures. The use of negative $\rho_{QV}$ can generate term structures with humped shapes for medium-leveraged firms and non-zero credit spread at short maturities of two to three years. These features are found by the empirical studies of Sarig and Warga (1989) for BBB-rated bonds. The use of positive $\rho_{QV}$ however cannot generate such features. The results based on negative $\rho_{QV}$ shows that negative correlation between the firm value and default barrier could be common in some medium-leveraged firms whose future leverage ratios may deviate from current levels over time. The derivation may happen when the firm value drops but external financing is needed, the firm would first seek debt funding according to the packing-order theory. The firm’s leverage ratio would thus increase accordingly.
Figure 6 shows the effect of the correlation $\rho_{QV}$ on the term structures of discount bonds issued by a highly-leveraged firm with $Q = 0.46$ and $\sigma_Q = 0.1$. The firm with $Q/V = 0.46$ can be considered as rated BB, i.e. a speculative grade. Similar to the results in Figure 5, Figure 6 shows that credit spreads are very sensitive to $\rho_{QV}$ and the credit spreads increase when $\rho_{QV}$ becomes negative. The depicted term structures with negative $\rho_{QV}$ exhibit steep upward slopes at short maturities and are downward-sloping for maturities longer than a few years. The shapes are consistent with the empirical findings of the downward-sloping term structures of highly-leveraged firms and speculative grade (Ba-rated and B-rated) bonds reported by Sarig and Warga (1989) and Fons (1994) respectively. This downward slope is generated by high initial default probabilities which are expected to decrease over time as the firm survives. However, the term structures based on positive $\rho_{QV}$ do not match with the empirical observations. The results based on negative $\rho_{QV}$ show that negative correlation between the firm value and default barrier could also happen in highly-leveraged firms which may first seek debt funding if external financing is needed according to the packing-order theory. Their leverage ratios would thus deviate from current levels over time. Figure 6 illustrate that the sensitivity of credit spreads to $\rho_{QV}$ appears to be reduced with the increase in time to maturity. This demonstrates that the correlation between the firm value and default barrier has larger impact on the default risk of speculative-graded bonds at short maturities than at long maturities.

In summary, the numerical results show that the model incorporating a stochastic default barrier with appropriate $\rho_{QV}$ (such as $\rho_{QV} = -0.3$ used in the above analysis) gives the basic shapes of the term structures of credit spreads similar to some empirical findings for similarly rated corporate bonds (i.e. AA, BBB and BB).
packing-order theory of firms’ capital structures could explain the use of negative $\rho_{QV}$.

The prediction by the proposed model using high $\sigma_Q$ for a medium- or highly-leveraged firm that credit spreads are non-zero at short maturities of two to three years is consistent with empirical findings. On the other hand, the use of low $\sigma_Q$ for a low-leveraged firm can generate the term structures whose shapes are consistent with those observed in Aa-rated bonds.

4. Conclusion

This paper develops a three-factor corporate bond valuation model that incorporates a stochastic default barrier. The model is in line with the packing-order theory of firms’ capital structures. As the default barrier is considered to be the bond issuer’s liability, bonds default when the bond issuer’s liability-to-asset ratio increases above a predefined level upon default. The closed-form solution of the corporate bond price is derived to obtain credit spreads. The credit spread analysis shows that the dynamics of the stochastic default barrier has material impact on the credit spreads of corporate bonds. The use of stochastic default barriers is capable of producing term structures of credit spreads that are consistent with some empirical findings. The model could therefore provide new insight for future research on corporate bonds analysis and credit risk modelling. More detailed empirical comparisons between the actual credit spreads and the model credit spreads are left to future research.

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