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PHYS 3202 Set 3 Suggested Solution

No

Date 5-2-12

1. (a) no. of Si atoms per unit cell $= \frac{1}{8} \times 8 + \frac{1}{2} \times 6 + 4$

$= 8$

(b) No. of Ga atoms per unit cell $= 4$

No. of As atoms per unit cell $= \frac{1}{8} \times 8 + \frac{1}{2} \times 6 = 4$

(c) $$\rho = \frac{\text{Total mass of atoms in a unit cell}}{a^3}$$

$$= \frac{4(m_{Ga} + m_{As})}{a^3}$$

$$\Rightarrow a = \left(\frac{4(m_{Ga} + m_{As})}{\rho}\right)^{\frac{1}{3}} = \left(\frac{4(\text{molar mass of Ga} + \text{molar mass of As})}{N_A \, \rho}\right)^{\frac{1}{3}}$$

N_A ← no. of particles per mole.

$$= \left(\frac{4(69.723 + 74.923)\ \mathrm{g\ mol^{-1}}}{(6.022 \times 10^{23}\ \mathrm{mol^{-1}})(5.3176 \times 10^{6}\ \mathrm{g\ m^{-3}})}\right)^{\frac{1}{3}}$$

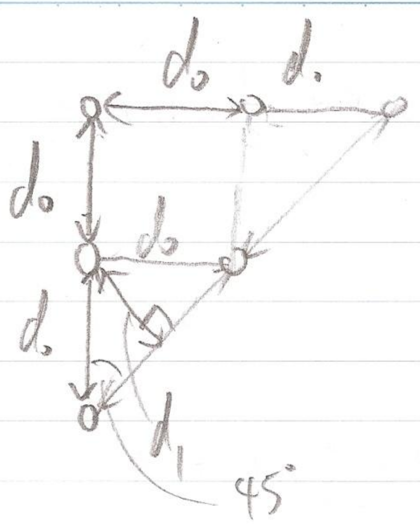
$= 5.65 \times 10^{-10}$ m

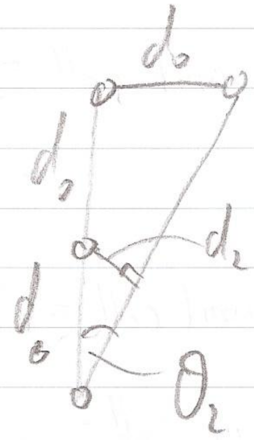
$= 5.65$ Å

Alternatively, if we take $m_{Ga} = 69.723\,u$, $m_{As} = 74.923\,u$, where $u = 1.66 \times 10^{-27}$ kg

Then $$a = \left(\frac{4(69.723 + 74.923)\,u}{(5.3176 \times 10^{6} \times 10^{-3})\ \mathrm{kg\ m^{-3}}}\right)^{\frac{1}{3}} = 5.65 \text{ Å}$$

2. (a)





$$d_1 = d_0 \sin 45° = \frac{d_0}{\sqrt{2}}$$

$$\sin \theta_2 = \frac{1}{\sqrt{5}}$$

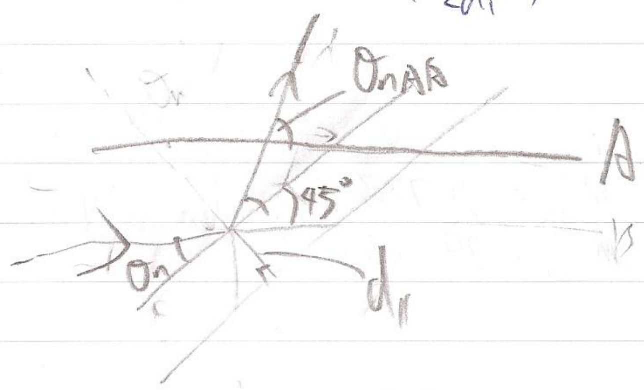
$$\therefore \quad d_2 = d_0 \sin \theta_2 = \frac{d_0}{\sqrt{5}}$$

(b) $2 d_1 \sin \theta_n = n\lambda$, $d_1 = \frac{4}{\sqrt{2}} = 2.83$ Å , $\lambda = 0.626$ Å

$$\Rightarrow \theta_1 = \sin^{-1}\left(\frac{\lambda}{2d_1}\right) = \sin^{-1}\left(\frac{0.626}{2(2.83)}\right) = 6.5°$$

$$\theta_2 = \sin^{-1}\left(\frac{2\lambda}{2d_1}\right) = \sin^{-1}\left(\frac{0.626}{2.83}\right) = 12.8°$$

$$\theta_3 = \sin^{-1}\left(\frac{3\lambda}{2d_1}\right) = \sin^{-1}\left(3 \cdot \frac{0.626}{2(2.83)}\right) = 19.4°$$



$$\theta_{nAA} = \theta_n + 45°$$

$$\Rightarrow \theta_{1AA} = 51.5°$$
$$\theta_{2AA} = 57.8°$$
$$\theta_{3AA} = 64.4°$$

3. (a) $eV_A = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2}$

$\Rightarrow \lambda = \frac{h}{\sqrt{2meV_A}}$

(b) $2d\sin\theta = \lambda$

For small θ, $\sin\theta \approx \theta$

where $2\theta \approx \frac{D}{2L}$,

and $\lambda = \frac{h}{\sqrt{2meV_A}}$,

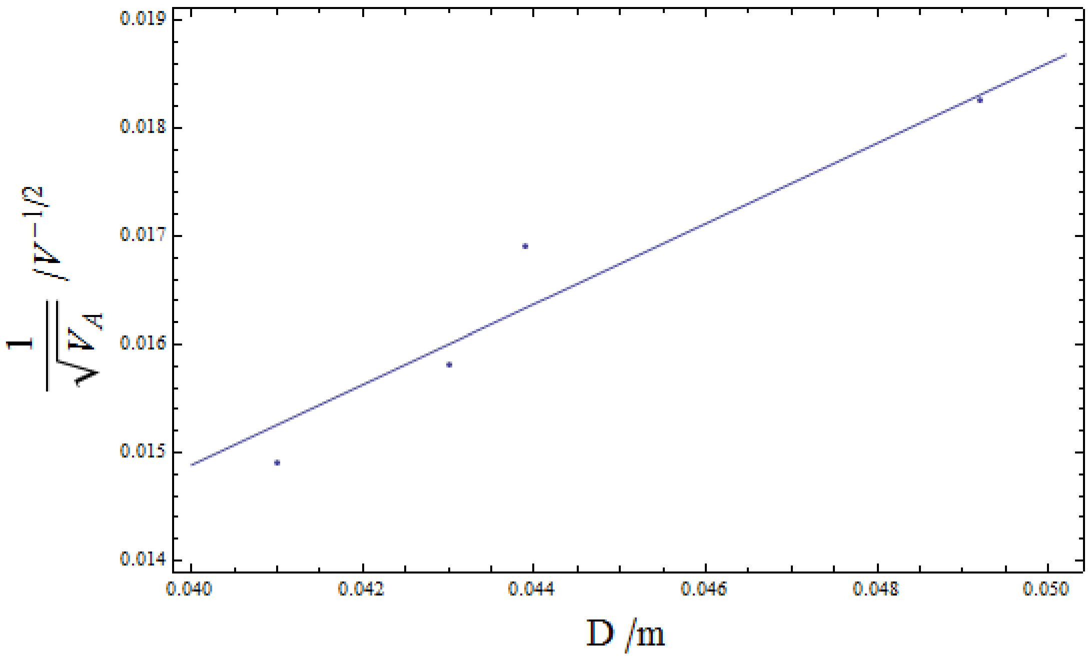
$\therefore 2d\sin\theta \approx \frac{Dd}{2L} = \frac{h}{\sqrt{2meV_A}}$

$\Rightarrow D = \frac{2hL}{d\sqrt{2me}} \frac{1}{\sqrt{V_A}} \Rightarrow \frac{1}{\sqrt{V_A}} = \frac{d\sqrt{2me}}{2hL} D$

(c) Slope of the curve of $\frac{1}{\sqrt{V_A}}$ v.s. D $= 0.372\ V^{-\frac{1}{2}} m^{-1}$

$\therefore 0.372 = \frac{d\sqrt{2me}}{2hL} = \frac{d\sqrt{(9.11\times10^{-31})(1.6\times10^{-19})}}{\sqrt{2}(6.63\times10^{-34})(0.130)}$

$\Rightarrow d = 1.19\ \mathring{A}$



The line is the best fit line: y = 0.372 x,
where y = 1/Sqrt[V_A], x = D.

Note that the fitted curve should pass through the origin.