

PHYS 3202: Problem set 5

Due: February 24, 2012 (Friday), 5:30 pm, SC213

(Hall Effect in semiconductor)

(In Ex Classes, TA will explain Appendix 8. But the derivation is for low field approximation.)

Preparation: Review the principle of Hall Effect in metal as taught in PHY 2822 & EM courses.

The Hall Effect is used to measure the charge carrier concentrations in a sample.

During the measurement, a uniform current (I) is passing through a thin plate of sample and a magnetic field (\vec{B}) is applied **perpendicular** to the plate. A voltage difference V_{ab} (called **Hall voltage**) proportional to B and I appears on the sample edges as shown in Fig. 1. An extra electric field is induced in \hat{y} direction. Here $\vec{B} = B\hat{z}$ is a uniform magnetic field.

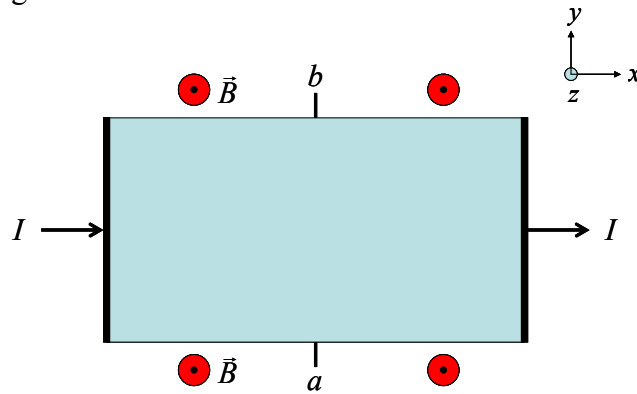


Fig. 1 Hall effect

The problem set:

(A) Consider a thin **metal** plate.

(i) Show that if $B = 0$, the drift velocity \vec{v}_e is related to the electric force on a conduction electron:

$$\vec{F} = -e\vec{E} = \frac{e\vec{v}_e}{\mu_e} \quad (5 \text{ marks})$$

Hint: $\vec{J}_e = -ne\vec{v}_e = \sigma\vec{E}$

(ii) For $B \neq 0$, write down a similar expression. (5 marks)

Hint: It is the Lorentz force on the electron.

(iii) Find the **Hall coefficient** R_H (defined as $E_y = R_H B J_x$). (5 marks)

Hint: For metal in equilibrium, \vec{v}_e is along \hat{x} axis.

(B) Consider a thin **semiconductor** plate with electron concentration n in conduction band and hole concentration p in valence band. The mobility of electrons is μ_e and the mobility of holes is μ_h .

A uniform current (I) is passing through the thin sample and a uniform magnetic field (\vec{B}) is applied perpendicular to the plate. An extra electric field is induced in \hat{y} direction.

(i) In equilibrium, \vec{v}_e has both x - and y -components ($\vec{v}_e = v_{ex}\hat{x} + v_{ey}\hat{y}$).

(For metal in equilibrium, \vec{v}_e is along the \hat{x} axis.)

Write down an expression to relate the drift velocity \vec{v}_e to the Lorentz force on the electron.

Show that the current density due to electrons in conduction band is

$$\vec{J}_e = J_{ex}\hat{x} + J_{ey}\hat{y}$$

where $J_{ex} = \sigma_{exx}E_x + \sigma_{exy}E_y$,

$$J_{ey} = \sigma_{eyx}E_x + \sigma_{eyy}E_y$$

$$\vec{E} = E_x\hat{x} + E_y\hat{y} \text{ is the electric field in the sample,}$$

$$\sigma_{\text{exx}} = \frac{ne\mu_e}{1 + (\mu_e B)^2} \text{ and}$$

$$\sigma_{\text{exy}} = -\sigma_{\text{eyx}} = \frac{-ne\mu_e^2 B}{1 + (\mu_e B)^2}. \quad (15 \text{ marks})$$

(ii) The current density due to holes in the valence band can be found in a similar way:

$$J_{hx} = \sigma_{hxx} E_x + \sigma_{hxy} E_y$$

$$J_{hy} = \sigma_{hyx} E_x + \sigma_{hyy} E_y$$

Find the expressions for σ_{hxx} and σ_{hxy} . (15 marks)

(iii) Then find the total current density

$$\vec{J} = J_x \hat{x} + J_y \hat{y}$$

where $J_x = J_{ex} + J_{hx}$ & $J_y = J_{ey} + J_{hy}$.

(iv) In equilibrium, $J_y = 0$. Based on this, show that the Hall coefficient is

$$R_H = \frac{1}{B} \frac{\sigma_{\text{exy}} + \sigma_{\text{hxy}}}{[\sigma_{\text{exx}} + \sigma_{\text{hxx}}]^2 + [\sigma_{\text{exy}} + \sigma_{\text{hxy}}]^2} \quad (10 \text{ marks})$$

(v) Draw a simply diagram to indicate drift velocity directions of charge carriers (electrons & holes). (5 marks)

(vi) Show that **for low magnetic field** ($\mu_e B \ll 1$ & $\mu_h B \ll 1$), the Hall coefficient is

$$R_H = \frac{p\mu_h^2 - n\mu_e^2}{e(p\mu_h + n\mu_e)^2} \quad (10 \text{ marks})$$

It is the formula derived in Appendix 8.

(vii) Consider a **silicon** sample **at room temperature** ($n_i = 1.5 \times 10^{10} \text{ cm}^{-3}$). (30 marks)

It is doped with 10^{16} cm^{-3} acceptors.

(a) Calculate the Hall coefficient if you consider only the majority charge carrier (holes in this case). (5 marks)

(b) Calculate the Hall coefficient if we consider both electrons and holes. The mobility:

$$\mu_e = 1350 \text{ cm}^2 \text{V}^{-1} \text{s}^{-1} \text{ and } \mu_h = 450 \text{ cm}^2 \text{V}^{-1} \text{s}^{-1}. \quad (10 \text{ marks})$$

(c) Calculate the conductivity using the equation $\sigma = ne\mu_e + pe\mu_h$. (5 marks)

(d) Calculate $E_F - E_i$. (10 marks)