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The Chinese University of Hong Kong

二〇一〇至二〇一一年度下學期科目考試
Course Examination 2nd Term, 2010–11

Course Code & Title 科目編號及名稱： PHYS 3202 Quantum Physics II

Time allowed 時間： 2 hours 小時 30 minutes 分鐘

Student I.D. No. 學生編號： Seat No. 座號：

Answer **ALL** questions.

1. Radioactive materials exist even inside our body. One example is ${}^{40}_{19}\mathrm{K}$. (20 marks)

Fig. 1 shows the decay scheme of ${}^{40}_{19}\mathrm{K}$. It has three beta-decay branches (**a**, **b** and **c**).

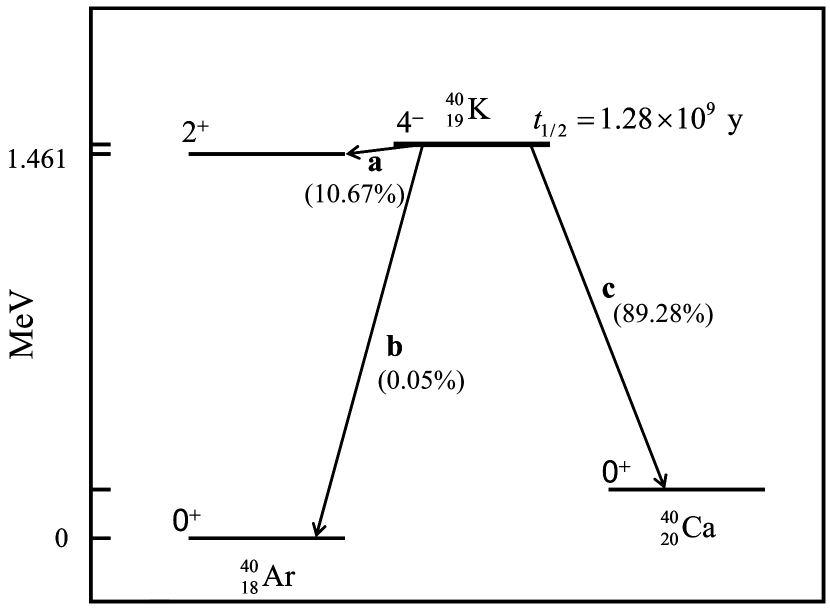

Fig. 1 Energy level diagrams for decay of ${}^{40}_{19}\mathrm{K}$.

(a) Identify each decay branch as either β^-, β^+ and/or electron capture. (2 marks)

(b) For each β^- decay branch, calculate the end-point energy of the emitted particles. (8 marks)

For each β^+ decay branch, calculate the end-point energy of the emitted particles.

For each electron capture decay branch, calculate the energy of the emitted particle.

(c) A ${}^{40}_{19}\mathrm{K}$ source also emits gamma photons. What are their energies? (1 mark)

(d) It is known that there are about 140 g of potassium in a person weighing 70 kg.

Calculate the gamma emission rate (activity) of this person. (7 marks)

(e) The ${}^{40}_{19}\mathrm{K}$ decays have been used to investigate the interior of our Earth. What particles from the decays should be studied? (2 marks)

Given: Molar mass of potassium = 39.10 g/mole.

Natural potassium contains 93.258% ${}^{39}_{19}\mathrm{K}$, 0.012% ${}^{40}_{19}\mathrm{K}$ and 6.730% ${}^{41}_{19}\mathrm{K}$.

Atomic masses: $M({}^{40}\mathrm{K}) = 39.963999$ u, $M({}^{40}\mathrm{Ar}) = 39.962384$ u, $M({}^{40}\mathrm{Ca}) = 39.962591$ u.

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2. (Superconductivity) (20 marks)

(a) Kamerlingh Onnes failed to construct a superconducting solenoid for 10 T magnetic field. Why? (2 marks)

(b) A ring made of Type I superconductor was used to illustrate perfect conductivity. The ring was first cooled from room temperature to a temperature close to 0 K in a uniform magnetic field $\vec{H}$. Then the applied field $\vec{H}$ is completely removed and a persistent current (I) is induced in the ring.

(i) Describe where the current is distributed. (2 marks)

(ii) The magnetic flux through the ring is quantized. The smallest flux is called flux quantum. What is the value of this flux quantum? (3 marks)

(iii) If now we reapply a magnetic field equal to $\frac{1}{2}\vec{H}$, what is the new current? (2 marks)

(iv) If now we reapply a magnetic field equal to $\vec{H}$, what is the new current? (2 marks)

(c) If this superconductor has a superconducting energy gap $2\Delta = 1\ \text{meV}$, estimate its transition temperature. (3 marks)

(d) The superconducting energy gap can be determined by measuring the *I-V* characteristics of an SIN tunneling junction (S: superconductor, I: insulator and N: normal metal). Sketch the *I-V* curve to indicate how this can be done. Explain with the help of an energy band diagram. (6 marks)

3. (Particle physics) (20 marks)

(a) Consider the reaction $\pi^- + \text{p} \rightarrow \text{n} + \pi^0$.

If π^- and p were initially at rest, determine the energy of the neutron.

The masses are given: $m_{\pi^0} = 135.0\ \text{MeV/c}^2$ and $m_{\pi^-} = 139.6\ \text{MeV/c}^2$.

Hint: You can use non-relativistic kinetic energies. (6 marks)

(b) For each of the following decay processes, find the interaction (weak, strong or electromagnetic) responsible for the decay:

(i) $\mu^- \rightarrow \text{e}^- + \bar{\nu}_e + \nu_\mu$, (ii) $\pi^0 \rightarrow \gamma + \gamma$,

(iii) $\rho^0 \rightarrow \pi^+ + \pi^-$, (iv) ${}^{241}_{95}\text{Am} \rightarrow {}^{237}_{93}\text{Np} + \alpha$ (4 marks)

(c) Recently, anti-α particles ($\bar{\alpha}$) have been discovered in collision experiments in Brookhaven's Relativistic Heavy-Ion Collider.

(i) What is the quark content of $\bar{\alpha}$? (2 marks)

(ii) Suppose you can have a beam of anti-α particles with particle energy of 5 MeV to bombard a thin gold foil and perform a Rutherford scattering experiment. Describe what may be your result? (4 marks)

(d) Is J/ψ particle a (i) lepton, (ii) baryon, or (iii) meson? (2 marks)

(e) What is the major origin of the proton mass? Proton mass: $m_p = 938.3\ \text{MeV/c}^2$.

Quark masses: $m_u \approx 3\ \text{MeV/c}^2$ and $m_d \approx 6\ \text{MeV/c}^2$. (2 marks)

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4. (Deuteron) (20 marks)

(a) The binding energy (B) of deuteron (d) can be measured by bombarding stationary deuterons with photons: $\gamma + \mathrm{d} \to \mathrm{n} + \mathrm{p}$. Calculate the threshold energy for this reaction.

Given: deuteron mass $m_d = 2.013553\ \mathrm{u}$, neutron mass $m_n = 1.008665\ \mathrm{u}$ and proton mass $m_p = 1.007276\ \mathrm{u}$. (4 marks)

(b) How many bound states are there for deuteron? (2 marks)

(c) The time-independent Schrödinger equation for deuteron can be solved if we assume a spherical potential well for the nuclear force:

$$U(r) = \begin{cases} -V_0 & \text{for } r < b \\ 0 & \text{for } r > b \end{cases}$$

where b is the range of nuclear force.

The deuteron wavefunction is expressed as $\psi \propto \phi(r) Y_{\ell m}(\theta, \varphi)$,

where $$-\frac{\hbar^2}{2\mu}\frac{d^2(r\phi)}{dr^2} + \left(\frac{\ell(\ell+1)\hbar^2}{2\mu r^2} + U(r)\right)(r\phi) = \varepsilon(r\phi) \qquad [1]$$

and $\mu \approx \frac{m_n}{2} \approx \frac{m_p}{2}$.

(i) The magnetic moments of d, n and p are $\mu_d = 0.857411\ \mu_N$, $\mu_p = 2.79271\ \mu_N$ and $\mu_n = -1.91315\ \mu_N$. The deuteron spin is $J = 1$. Based on this information, can you find the value of ℓ? (2 marks)

(ii) To solve equation [1], take $u \equiv r\phi$. State the boundary conditions for u. (2 marks)

(iii) The wavefunction for $r < b$ is

$$u = A \sin Kr$$

where $K = \sqrt{\frac{2\mu(V_0 + \varepsilon)}{\hbar^2}}$ and A is a normalization constant.

What is the suitable form of u for $r > b$? (4 marks)

(iv) Find the expression relating b and B to V_0. (6 marks)

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5. (Solar neutrinos) (20 marks)

Davis measured solar neutrinos for 25 years using a tank of C_2Cl_4 liquid. Natural chlorine has 75.77% ${}^{35}_{17}\mathrm{Cl}$ and 24.23% ${}^{37}_{17}\mathrm{Cl}$. The reason he used is

$$\nu_e + {}^{37}_{17}\mathrm{Cl} \rightarrow {}^{37}_{18}\mathrm{Ar}^* + e^-$$

as illustrated in Fig. 2

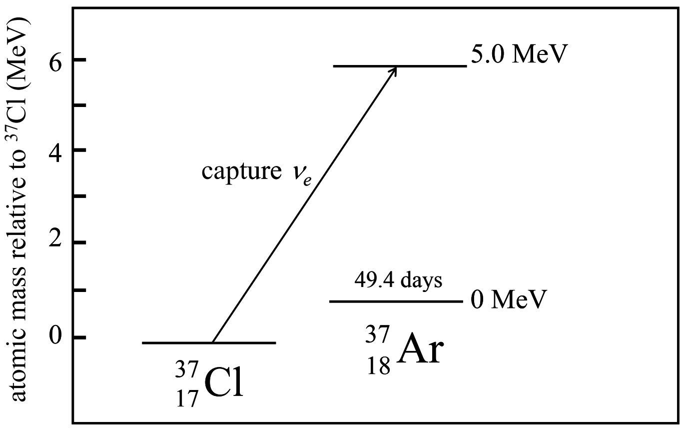

Fig. 2 Energy level diagrams for solar neutrino experiment.

(a) Explain briefly why chlorine is a good choice for this neutrino experiment. (2 marks)

(b) How did Davis collect the ${}^{37}_{18}\mathrm{Ar}$ atoms produced in the reactions? (2 marks)

(c) Explain how he measured the number of ${}^{37}_{18}\mathrm{Ar}$ atoms. (3 marks)

(d) The setup was 1500 m underground in a gold mine. Why is it necessary? (1 mark)

(e) Is his result consistent with the theoretical prediction? If not, is there any explanation of the discrepancy? (2 marks)

(f) Write down the net fusion process in the sun? (3 marks)

(Hint: $4\,{}^{1}\mathrm{H} \rightarrow {}^{4}\mathrm{He} + \ldots$)

(g) If the reaction cross-section averaged over the useful energy range is $1\times10^{-42}\ \mathrm{cm}^2$ and the production of ${}^{37}\mathrm{Ar}$ is one every two days, calculate the flux of detectable neutrinos. (7 marks)

Given: Davis used 380 m^3 C_2Cl_4 (density $= 1.62\ \mathrm{g/cm}^3$ and molar mass $= 165.83$ g/mole).

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PHYSICAL CONSTANTS & CONVERSION FACTORS

Named Constants

Atomic mass unit: $1\ \text{u} = \frac{1}{12}\text{m}(^{12}\text{C atom})$
$= 1.66\times10^{-27}\,\text{kg}$
$= 931.5\ \text{MeV}/c^2$

Avogadro's constant: $N_\text{A} = 6.02\times10^{23}$ particles/mole

Bohr radius: $a_\text{B} = \hbar^2/(ke^2m_\text{e})$
$= 5.29\times10^{-11}\,\text{m}$

Boltzmann's constant: $k_\text{B} = 8.62\times10^{-5}\,\text{eV/K}$
$= 1.38\times10^{-23}\,\text{J/K}$

Coulomb force constant: $k = 1/(4\pi\varepsilon_0) = \mu_0 c^2/(4\pi)$
$= 8.99\times10^{9}\,\text{N}\cdot\text{m}^2/\text{C}^2$

Electron Compton wavelength: $\lambda_\text{c} = h/(m_\text{e}c)$
$= 2.43\times10^{-12}\,\text{m}$

Electron volt: $1\ \text{eV} = 1.60\times10^{-19}\,\text{J}$

Elementary charge: $e = 1.60\times10^{-19}\,\text{C}$

Fine-structure constant: $\alpha = ke^2/(\hbar c)$
$= 7.30\times10^{-3} \approx 1/137$

Gas constant: $R = 8.31\ \text{J}/(\text{mole}\cdot\text{K})$
$= 0.0821\ \text{liter}\cdot\text{atm}/(\text{mole}\cdot\text{K})$

Gravitational constant: $G = 6.67\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2$

Mass of electron: $m_\text{e} = 5.485799110\times10^{-4}\,\text{u}$
$\approx 9.11\times10^{-31}\,\text{kg}$
$= 0.511\ \text{MeV}/c^2$

Mass of proton: $m_\text{p} = 1.00727646688\ \text{u}$
$\approx 1.673\times10^{-27}\,\text{kg}$
$= 938.3\ \text{MeV}/c^2$

Mass of neutron: $m_\text{n} = 1.00866491578\ \text{u}$
$\approx 1.675\times10^{-27}\,\text{kg}$
$= 939.6\ \text{MeV}/c^2$

Bohr magneton: $\mu_\text{B} = e\hbar/(2m_\text{e})$
$= 5.79\times10^{-5}\,\text{eV/T}$
$= 9.27\times10^{-24}\,\text{J/T}(\text{or A}\cdot\text{m}^2)$

Nuclear magneton: $\mu_\text{N} = e\hbar/(2m_\text{p})$
$= 3.15\times10^{-8}\,\text{eV/T}$
$= 5.05\times10^{-27}\,\text{J/T}$

Radioactive source activity: $1\ \text{Ci} = 3.7\times10^{10}\ \text{Bq}$

Permeability of vacuum: $\mu_0 = 4\pi\times10^{-7}\,\text{N/A}^2$
$= 1.26\times10^{-6}\,\text{N/A}^2$

Permittivity of vacuum: $\varepsilon_0 = 1/(\mu_0 c^2)$
$= 8.85\times10^{-12}\,\text{C}^2/(\text{N}\cdot\text{m}^2)$

Planck's constant: $h = 6.63\times10^{-34}\,\text{J}\cdot\text{s}$
$= 4.14\times10^{-15}\,\text{eV}\cdot\text{s}$
$\hbar = h/2\pi$
$= 1.05\times10^{-34}\,\text{J}\cdot\text{s}$
$= 6.58\times10^{-16}\,\text{eV}\cdot\text{s}$

Rydberg constant: $R = m_\text{e}k^2e^4/(4\pi c\hbar^3)$
$= 1.10\times10^{-2}\,\text{nm}^{-1}$

Rydberg energy: $E_\text{R} = hcR = m_\text{e}k^2e^4/(2\hbar^2)$
$= 13.6\ \text{eV}$

Speed of light in vacuum: $c = 3.00\times10^{8}\,\text{m/s}$

Useful Combinations

$hc = 1240\ \text{eV}\cdot\text{nm} = 1240\ \text{MeV}\cdot\text{fm}$

$\hbar c = 197\ \text{eV}\cdot\text{nm} = 197\ \text{MeV}\cdot\text{fm}$

$ke^2 = 1.44\ \text{eV}\cdot\text{nm} = 1.44\ \text{MeV}\cdot\text{fm}\,, \quad k = \frac{1}{4\pi\varepsilon_0}$

$N_\text{A}\times(1\ \text{u}) = 1\ \text{gram}$

$k_\text{B}T = 0.026\ \text{eV}$ at room temperature (300K)

Conversion Factors

Area: 1 barn = $10^{-28}\ \text{m}^2$

Energy: 1 cal = 4.184 J
1 eV = 1.60×10^{-19} J

Length: 1Å = 1 angstrom
= 10^{-10} m
1 ft = 30.48 cm
1 in = 2.54 cm
1 mi = 1609 m

Mass: 1 kg = 2.20 lb
1 MeV/c^2 = 1.07×10^{-3} u
= 1.78×10^{-30} kg

Momentum: 1 MeV/c = $5.34\times10^{-22}\,\text{kg}\cdot\text{m/s}$