An example on error analysis

Consider a **free fall experiment**

Objective: measurement of *g*

Theory: $d = \frac{1}{2} g t^2$

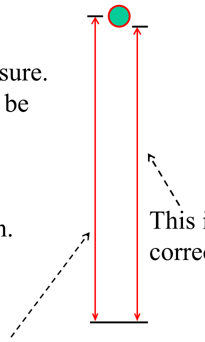
(d: height; t: falling time)

Simplification:
we neglect air friction.

Equipment:
for *d*: meter stick or measuring tape
for *t*: stopwatch or smart timer

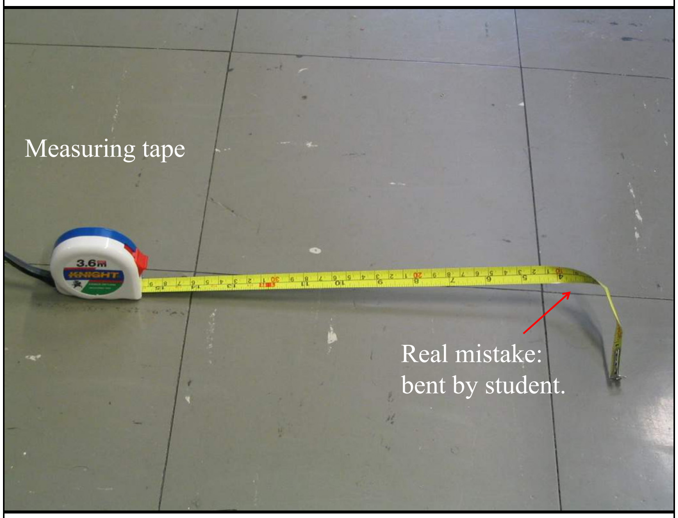
Measurement of *d:*

We measure the height with a meter stick or measuring tape.
These are analog equipment
with instrument error: < 0.5 mm
But you need to judge where we measure.
This human error (uncertainty) could be
~ 3 mm.
∴ **measurement error** of this datum
= ± 3 mm.
The judgment is subject to fluctuation.
So it is a random error.



This is the correct *d*.

If you take this height as *d*, then you have a systematic error & it is a real mistake.



Measurement of *t*:
We may use a stopwatch.
It is a digital equipment with instrument error < 0.01 s.

But you need to judge when we push the button.
∴ There is human uncertainty (i,e., your reaction time)
≈ 0.1 – 0.2 s.
You push is always delayed & thus this is a systematic error, which is also subject to random fluctuation.

To start stopwatch, we have systematic error + random error
to stop stopwatch, we have systematic error + random error.
⇒ The measured *t* has basically only the random error.

In order to avoid human uncertainty, we use a smart timer (a digital device) to measure *t* for this experiment.

The start & stop of the smart timer is triggered by a simple mechanism. (Its detail is described in the user manual of the smart timer available for download on PHY 2811 Web Page.)

The smart timer has
accuracy = ± (1 least significant digit).

This is a **random error**.

The timer was calibrated by the manufacture so the systematic error is negligible.

Human error ≈ 0

Suppose the reading is 0.4723 s.
↑ the least significant digit

Then the measurement error = ± 0.0001 s.
Your data should be 0.4723 ± 0.0001 s.

This experiment is separated into two parts.

Part I: Distribution of data

Here we keep a constant **falling height**:
$d = 1.300 \pm 0.003$ m

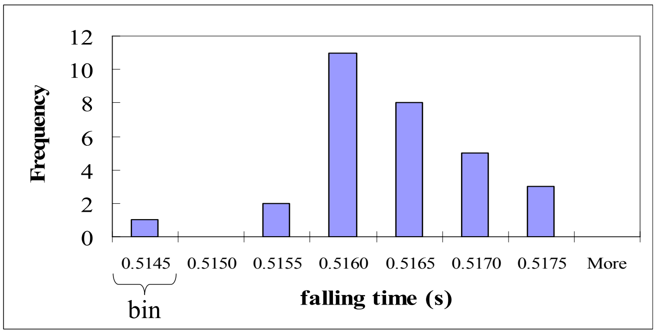
& repeat measurements of ***t***.

We obtain a distribution of data:
the falling tine: $t = t_1, t_2, t_3, \ldots, t_n$

We can analyze data by Excel
as described in my Short Notes on Error Analysis.

The data file is available for download on PHY2811 Web Page.

Plot histogram of data by Excel

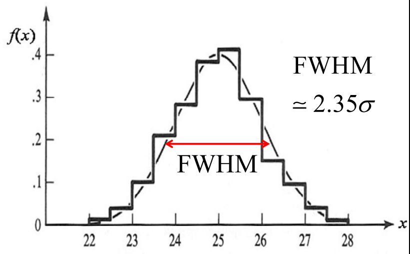


The 1st bin 0.5145 is for data ranging from 0.5141 to 0.5145.
The 2nd bin 0.5150 is from 0.5146 to 0.5150.

It should be a normal distribution if *n* is very large.

Normal/Gaussian distribution

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\bar{x})^2/2\sigma^2}$$



$$\sigma = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

(true standard deviation)

$$\bar{x} = \left(\sum_{i=1}^{n} x_i\right)/n$$

What is experimental error?

Measure one data set (x_1, x_2, x_3 …,).
Present your result as

$\bar{x} \pm \delta x$

standard error $\quad \delta x = \sqrt{\dfrac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n(n-1)}}$

Meaning of δx :

If a data set is measured again,
there is 68% chance to get a new value of $\bar{x}$
falling within the interval: $\bar{x} \pm \delta x$

Present result for the free fall experiment:

$t = \bar{t} \pm \delta t$ ⟶ standard error $\quad \delta t = \sqrt{\dfrac{\sum_{i=1}^{n}(t_i - \bar{t})^2}{n(n-1)}}$

Result for this set of data by **Excel** analysis:

$t = 0.5161 \pm 0.0001$ s

You can use other format: $t = 0.51610 \pm 0.00011$ s

or $\quad t = 0.5161 \pm 0.02\%$ s

↑ not recommended

Note:

(1) **Keep only 1 or 2 significant figure(s) for error.**

(2) The datum should have significant figures consistent with its error.

Part II: Free fall experiment

To verify $\quad d = \frac{1}{2}gt^2$,

we measure $\bar{t}$
as a function of d
& the result is listed in Table 1.

Table 1. Free fall data

d (m)	Average t (s)
1.300 ± 0.003	0.5161 ± 0.0001
1.250	0.5058
1.200	0.4960
1.150	0.4850

- should have correct significant figures
- should cover a large range

Now we verify the theory: $d = \frac{1}{2}gt^2$.

Do not plot d vs t.
because it is not a linear graph.

Always try to plot a linear graph, e.g., d vs t^2

(Why not t^2 vs d ?)

& perform a **linear least square fit** to the linear graph.

Principle of linear least square fit:

Given: n data points: (x_i, y_i), $i = 1, 2, \ldots n$.

Try to fit $\quad y = ax + b$

Assumptions: see Short Notes on Error Analysis.

a & b are obtained by minimizing

$$S = \frac{1}{n}\sum_{i=1}^{n}(y_i - ax_i - b)^2$$

This is done by Excel & then we obtain

$a \pm \delta a$

$b \pm \delta b$

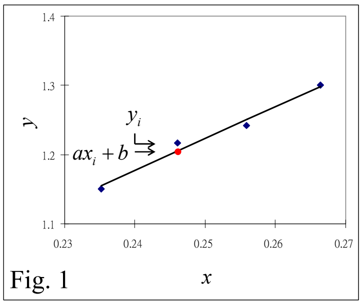


Fig. 1

Data analysis: to find g

The slope of the line in Fig. 1 is s = 4.84 ± 0.06 m/s².

by regression (迴歸)

According to $\quad d = \frac{1}{2}gt^2$,

the slope: $\quad s = \frac{1}{2}g$

$\therefore \quad g = 2(4.84)$

$= 9.68$ m/s²

We need to calculate error before we can determine the correct significant figures

Error estimation

$g = 2s$

By propagation of error:

$$\frac{\delta g}{g} = \frac{\delta s}{s}$$

$= 1.2\%$

by regression $\delta s = 0.06$

slope $s = 4.84$

Significant figures:

$\delta g = 9.68 \times 1.2\%$

Hence g = 9.68 ± 0.12 m/s²

9.7 ± 0.1 m/s²

9.68 m/s² ± 1.2%

Compare the experimental result with expected value

The expected value of g in Hong Kong is 9.79 m/s².

Difference $= \dfrac{9.79\text{-}9.68}{9.79} \times 100\%$

$= 1\%$

Conclusion:

Free fall equation is verified.
The measured value of g agrees with the expected value within the experimental error (±1.2%).
The discrepancy is 1%.

(Data Sheets are available for download in PHY 2811 Course Page).
You need to calculate error when you see ± sign in the Data Sheet.

Appendix: data sheet

Name :________________

Date :________________

Part I: Distribution of measurement

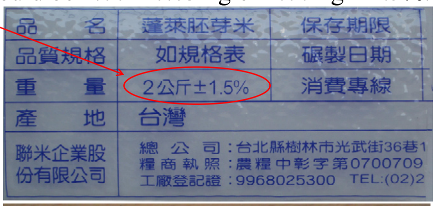
d = __________ ± __________ m

Table 1. Distribution of free fall time t

±				

$\bar{t}$ = sec
standard error = ± sec
Result: t = ________ ± ________ sec

More examples: products from Taiwan:
Bad example: It should be 2.00 ± 0.03 kg or 2.00 kg ± 1.5%.



Good example:

More example: 25.5°C ???

- In 2006, the Electrical and Mechanical Services Department of Hong Kong Government suggested a suitable room temperature: 25.5°C.
- However the air conditioners in Hong Kong are usually controlled in on-off mode & thus the temperature fluctuation is quite large, at least > 1°C.
- Usually only integer temperature can be set and read.
- To make sense, the suggested temperature (if correct) should be either 25 or 26°C.
- Temperature is not the only parameter that affects our comfort.