PHY 2811 (2009)

Solution of Exp. 6 prelab questions

(1) (Done by Lee CK)

For simplicity, set $r = R$.

$\Gamma_f \equiv$ torque due to friction on the axle of pulley

$r_p \equiv$ radius of pulley

$I_p \equiv$ moment of inertia of pulley

$f \equiv$ friction between the cylinder and table

Then

$$2mg - 2T_1 = 2ma_m \qquad [1]$$

$$2T_2 - f = M \cdot a_c \qquad [2]$$

$$(T_1 - T_2)r_p - \Gamma_f = I_p \ddot{\theta}_p \qquad [3]$$

$$(T_1 - T_2)r_p - \Gamma_f = I_p \frac{a_m}{r_p} \qquad [4]$$

Eq. [1] + Eq. [2]: $2mg - 2(T_1 - T_2) - f = 2m \cdot a_m + M \cdot a_c$ [5]

Substituting Eq. [4] into Eq. [5]:

$$2mg - \frac{2}{r_p}\left(I_p \frac{a_m}{r_p} + \Gamma_f \right) - f = 2ma_m + Ma_c$$

$$f = 2mg - \frac{2\Gamma_f}{r_p} - a_m\left(2m + \frac{2I_p}{r_p^2} \right) - Ma_c$$

$$= 2mg - \frac{2\Gamma_f}{r_p} - \left[M + 2\left(1 + \frac{r}{R} \right)\left(m + \frac{I_p}{r_p^2} \right) \right] a_c$$

For our pulley, $I_p = 1.23 \times 10^{-5}\,\text{kg} \cdot \text{m}^2$, $r_p = 0.031$ m & $\Gamma_f \approx 1.12 \times 10^{-4}$ N · m.

(2) $2m/(3M + 4m) = 0.005$